

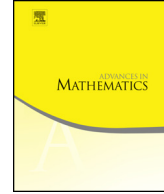


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# Variation of the Nazarov–Sodin constant for random plane waves and arithmetic random waves



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## ABSTRACT

This is a manuscript containing the full proofs of results announced in [10], together with some recent updates. We prove that the *Nazarov–Sodin constant*, which up to a natural scaling gives the leading order growth for the expected number of nodal components of a random Gaussian field, genuinely depends on the field. We then infer the same for “arithmetic random waves”, i.e. random toral Laplace eigenfunctions.

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## 1. Introduction

For  $m \geq 2$ , let  $f : \mathbb{R}^m \rightarrow \mathbb{R}$  be a *stationary* centred Gaussian random field, and  $r_f : \mathbb{R}^m \rightarrow \mathbb{R}$  its covariance function

$$r_f(x) = \mathbb{E}[f(0) \cdot f(x)] = \mathbb{E}[f(y) \cdot f(x + y)].$$

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Given such an  $f$ , let  $\rho$  denote its spectral measure, i.e. the Fourier transform of  $r_f$  (assumed to be a probability measure); note that prescribing  $\rho$  uniquely defines  $f$  by Kolmogorov’s Theorem (cf. [6, Chapter 3.3]). In what follows we often allow for  $\rho$  to vary; it will be convenient to let  $f_\rho$  denote a random field with spectral measure  $\rho$ . We further assume that a.s.  $f_\rho$  is sufficiently smooth, and that the distribution of  $f$  and its derivatives is non-degenerate in an appropriate sense (a condition on the support of  $\rho$ ).

*1.1. Nodal components and the Nazarov–Sodin constant*

Let  $\mathcal{N}(f_\rho; R)$  be the number of connected components of  $f_\rho^{-1}(0)$  in  $B_0(R)$  (the radius- $R$  ball centred at 0), usually referred to as the *nodal components* of  $f_\rho$ ;  $\mathcal{N}(f_\rho; R)$  is a random variable. Assuming further that  $f_\rho$  is ergodic (equivalently,  $\rho$  has no atoms), with non-degenerate gradient distribution (equivalent to  $\rho$  not being supported on a hyperplane passing through the origin), Nazarov and Sodin [21, Theorem 1] evaluated the expected number of nodal components of  $f_\rho$  to be asymptotic to

$$\mathbb{E}[\mathcal{N}(f_\rho; R)] = c_{NS}(\rho) \cdot \text{Vol}(B(1)) \cdot R^m + o(R^m), \tag{1.1}$$

where  $c_{NS}(\rho) \geq 0$  is a constant, subsequently referred to as the “Nazarov–Sodin constant” of  $f_\rho$ , and  $\text{Vol}(B(1))$  is the volume of the radius-1  $m$ -ball  $B(1) \subseteq \mathbb{R}^m$ . They also established convergence in mean, i.e., that

$$\mathbb{E} \left[ \left| \frac{\mathcal{N}(f_\rho; R)}{\text{Vol}(B(1)) \cdot R^m} - c_{NS}(\rho) \right| \right] \rightarrow 0; \tag{1.2}$$

it is a consequence of the assumed ergodicity of the underlying random field  $f_\rho$ . In this manuscript we will consider  $c_{NS}$  as a function of the spectral density  $\rho$ , without assuming that  $f_\rho$  is ergodic. To our best knowledge, the value of  $c_{NS}(\rho)$ , even for a single  $\rho$ , was not rigorously known heretofore.

For  $m = 2$ ,  $\rho = \rho_{S^1}$  the uniform measure on the unit circle  $S^1 \subseteq \mathbb{R}^2$  (i.e.  $d\rho = \frac{d\theta}{2\pi}$  on  $S^1$ , and vanishing outside the circle) the corresponding random field  $f_{\text{RWM}}$  is known as *random monochromatic wave*; according to Berry’s *Random Wave Model* [3],  $f_{\text{RWM}}$  serves as a universal model for Laplace eigenfunctions on generic surfaces in the high energy limit. The corresponding *universal* Nazarov–Sodin constant

$$c_{\text{RWM}} = c_{NS} \left( \frac{d\theta}{2\pi} \right) > 0 \tag{1.3}$$

was proven to be strictly positive [14]. Already in [4], Bogomolny and Schmit employed the percolation theory to predict its value, but recent numerics by Nastacescu [13], Konrad [8] and Beliaev–Kereta [2], consistently indicate a 4.5–6% deviation from these predictions.

More generally, let  $(\mathcal{M}^m, g)$  be a smooth compact Riemannian manifold of volume  $\text{Vol}(\mathcal{M})$ . Here the restriction of a fixed random field  $f : \mathcal{M} \rightarrow \mathbb{R}$  to growing domains, as

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