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Full extremal process, cluster law and freezing for the two-dimensional discrete Gaussian Free Field

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A B S T R A C T

We study the local structure of the extremal process associated with the Discrete Gaussian Free Field (DGFF) in scaled-up (square-)lattice versions of bounded open planar domains subject to mild regularity conditions on the boundary. We prove that, in the scaling limit, this process tends to a Cox process decorated by independent, correlated clusters whose distribution is completely characterized. As an application, we control the scaling limit of the discrete supercritical Liouville measure, extract a Poisson–Dirichlet statistics for the limit of the Gibbs measure associated with the DGFF and establish the "freezing phenomenon" conjectured to occur in the "glassy" phase. In addition, we prove a local limit theorem for the position and value of the absolute maximum. The proofs are based on a concentric, finite-range decomposition of the DGFF and entropic-repulsion arguments for an associated random walk. Although we naturally build on our earlier work on this problem, the methods developed here are largely independent.

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Contents

1. Introduction

Recent years have witnessed remarkable advances in the understanding of extreme values of the two-dimensional Discrete Gaussian Free Field (DGFF). This is a Gaussian process $\{h_x \colon x \in V\}$ in a proper subset V of the square lattice $\mathbb{Z}^2$ such that

$$E(h_x) = 0 \quad \text{and} \quad E(h_x\, h_y) = G^V(x,y), \tag{1.1}$$

where G^V denotes the Green function of the simple symmetric random walk in V killed upon exit from V. (We think of h_x as fixed to zero outside V.) Early efforts focused on