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New area-minimizing Lawson–Osserman cones

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ABSTRACT

It has been 40 years since Lawson and Osserman introduced the three minimal cones associated with Dirichlet problems in their 1977 Acta paper [13]. The first cone was shown area-minimizing by Harvey and Lawson in the celebrated paper [10]. In this paper, we confirm that the other two are also area-minimizing. In fact, we show that every Lawson–Osserman cone of type $(n, p, 2)$ constructed in [26] is area-minimizing.

Contents

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1. Introduction

Let $\Sigma \subset S^N \subset \mathbf{R}^{N+1}$ be an oriented closed submanifold (or rectifiable current) in the unit sphere. Then

$$\mathscr{C}\Sigma = \{tx : t \in [0, \infty) \text{ and } x \in \Sigma\}$$

is called the cone over Σ. We say $\mathscr{C}\Sigma$ is area-minimizing, if the truncated cone inside the unit ball has least area among all integral currents with boundary Σ.

The study of area-minimizing cones is a central topic in the geometric measure theory. By the well-known result of Federer (Theorem 5.4.3 in [5], also see Theorem 35.1 and Remark 34.6 (2) in Simon [19]) that a tangent cone at a point of an area-minimizing rectifiable current is itself area-minimizing, it is meaningful to explore the diversity of area-minimizing cones for better understandings about local behaviors of area-minimizing integral currents.

Area-minimizing cones also capture behaviors at infinity for area-minimizing surfaces in certain cases. The celebrated Bernstein problem stimulates the study on the nonexistence, the existence and the diversity of area-minimizing hypercones, e.g. [8,6,1,21,2,12,17,18,9,7,20,11]. In contrast, not quite much is known about area-minimizing cones of higher codimensions. Following [12], Cheng [4] found homogeneous area-minimizing cones of codimension two. Around the same time, Lawlor [11] developed a systematic method, called the curvature criterion, for determining whether a minimal cone is indeed area-minimizing, for instance, the classification of area-minimizing cones over products of spheres and the first examples of minimizing cones over nonorientable surfaces in the sense of mod 2.

Among others, three interesting non-parametric minimal cones were constructed by Lawson and Osserman [13] as follows. Let η, η' and η'' denote the (normalized) Hopf maps $S^{2^i-1} \to S^{2^{i-1}}$ for $i = 2,\ 3$ and 4. Then Lawson and Osserman considered the minimal embeddings

$$S^{2^i-1} \to S^{2^i+2^{i-1}}, \qquad x \mapsto \Big(\alpha_2 x, \beta_2\eta(x)\Big),\ \ \Big(\alpha_3 x, \beta_3\eta'(x)\Big),\ \ \Big(\alpha_4 x, \beta_4\eta''(x)\Big) \tag{1.1}$$

with $\alpha_i = \sqrt{\frac{4(2^{i-1}-1)}{3(2^i-1)}}$ and $\beta_i = \sqrt{\frac{2^i+1}{3(2^i-1)}}$ respectively. Over these minimal spheres, three minimal cones C_i for $i = 2,\ 3$ and 4 are then obtained. They, respectively, produce