Advances in Mathematics 329 (2018) 157–173

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Cluster values for algebras of analytic functions ☆

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ARTICLE INFO

Article history:
Received 19 September 2016
Received in revised form 11 August 2017
Accepted 15 August 2017
Available online xxxx
Communicated by C. Fefferman

MSC:
46J15
46E50
30H05

Keywords:
Cluster value problem
Corona Theorem
Ball algebra
Analytic functions of bounded type
Spectrum
Fiber

ABSTRACT

The *Cluster Value Theorem* is known for being a weak version of the classical *Corona Theorem*. Given a Banach space X, we study the *Cluster Value Problem* for the ball algebra $A_u(B_X)$, the Banach algebra of all uniformly continuous holomorphic functions on the unit ball B_X; and also for the Fréchet algebra $H_b(X)$ of holomorphic functions of bounded type on X (more generally, for $H_b(U)$, the algebra of holomorphic functions of bounded type on a given balanced open subset $U \subset X$). We show that Cluster Value Theorems hold for all of these algebras whenever the dual of X has the bounded approximation property. These results are an important advance in this problem, since the validity of these theorems was known only for trivial cases (where the spectrum is formed only by evaluation functionals) and for the infinite dimensional Hilbert space.

☆ This work was partially supported by projects CONICET PIP 11220130100329CO, ANPCyT PICT 2015-2299, ANPCyT PICT-2015-3085, ANPCyT PICT-2015-2224, UBACyT 20020130300057BA, UBACyT 20020130300052BA, UBACyT 20020130100474BA and MINECO and FEDER Project MTM2014-57838-C2-2-P.

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http://dx.doi.org/10.1016/j.aim.2017.08.030
0001-8708/

As a consequence, we obtain weak *analytic Nullstellensatz theorems* and several structural results for the spectrum of these algebras.

0. Introduction

In the 1940's a change of perspective in complex analysis emerged. S. Kakutani started a systematic study, from a Banach algebra point of view, of the space of bounded holomorphic functions on the open unit disk of the complex plane, $H^\infty(\mathbb{D})$.

A prominent group of mathematicians (Kaplansky, Wermer, Kakutani, Buck, Royden, Gleason, Arens and Hoffman) made several contributions in this line, and in 1961 published a very interesting paper [23] under the fictitious name of I.J. Schark. They showed that for each function $f \in H^\infty(\mathbb{D})$, the set of evaluations $\varphi(f)$ of elements on the maximal ideal space lying in the fiber (this will be described soon) over a point $z \in \partial\mathbb{D}$ coincides with the cluster set of f at z (i.e., the set of all limits of values of f along nets converging to z). This result, called the *Cluster Value Theorem*, was a sort of predecessor (and a weak version) of the famous Corona Theorem due to Carleson [13], which states that the characters given by evaluations on points of $\mathbb{D}$ are dense on the maximal ideal space of $H^\infty(\mathbb{D})$.

Several domains where the Corona Theorem fails are known [24]. Also it is unknown whether there is *any* domain in $\mathbb{C}^n$, for $n \geq 2$, for which the Corona Theorem holds. On the other hand, up to our knowledge, no domain is known for which the Cluster Value Theorem is not true.

In the context of infinite dimensional complex analysis, characterizing the cluster set is a non-trivial task, even for interior points. This fact led, in the last years, many mathematicians in the area to study the *Cluster Value Problem* for different algebras of analytic functions on infinite dimensional Banach spaces. Among their contributions, they showed positive results on the Cluster Value Theorem for the algebra $H^\infty(B_X)$ of bounded holomorphic functions on B_X, the open unit ball of the Banach space X [4,18,17]; the ball algebra $A_u(B_X)$ of all uniformly continuous holomorphic functions on B_X [4], and the algebra $H_b(X)$ of holomorphic functions of bounded type on X [5] (note that the first two, endowed with the norm $\|f\| = \sup_{x\in B_X} |f(x)|$ are Banach algebras and the last one, with the topology of uniform convergence on bounded sets, is a Fréchet algebra). These results were given for very specific Banach spaces. Namely, for the algebra $H^\infty(B_X)$, the known cases (besides finite dimensional spaces) are c_0 and $C(K)$, the space of null sequences and the space of continuous functions over a dispersed compact Hausdorff space K respectively [4,17]. For the algebras $A_u(B_X)$ and $H_b(X)$, the validity of the Cluster Value Theorem was known only for trivial cases (where the spectrum is formed only by evaluation functionals) and for the infinite dimensional Hilbert space [4,5]. Apart from these, there is (up to our knowledge) no general positive