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On the Bernoulli property for certain partially hyperbolic diffeomorphisms

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ABSTRACT

We address the classical problem of equivalence between Kolmogorov and Bernoulli property of smooth dynamical systems. In a natural class of volume preserving partially hyperbolic diffeomorphisms homotopic to Anosov ("derived from Anosov") on 3-torus, we prove that Kolmogorov and Bernoulli properties are equivalent.

In our approach, we propose to study the conditional measures of volume along central foliation to recover fine ergodic properties for partially hyperbolic diffeomorphisms. As an important consequence we obtain that there exists an almost everywhere conjugacy between any volume preserving derived from Anosov diffeomorphism of 3-torus and its linearization. Our results also hold in higher dimensional case when central bundle is one dimensional and stable and unstable foliations are quasi-isometric.

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1. Introduction

One of the main goals of smooth ergodic theory is to describe or understand the behavior of typical orbits $\{f^n(x)\}_{n\in\mathbb{Z}}$ of a given measure preserving diffeomorphism $f : M \to M$ on a Riemannian manifold M. Several topological and metric invariants were introduced in the theory in order to classify the dynamics of flows and diffeomorphisms by detecting, in some sense, the amount of chaoticity of a dynamical system. Two very effective and well-known invariants are the topological and metric entropy. If the metric entropy of a μ-measure preserving automorphism f is greater than the metric entropy of g with respect to a g-invariant measure ν, we can say that from the ergodic point of view the orbits of f have a richer behavior compared to the orbits of g.

In the seventies, D. Ornstein proved that the metric entropy is a complete invariant for the class of Bernoulli shifts, that is, two Bernoulli shifts with the same metric entropy are isomorphic [25,27]. Bernoulli shifts have a very strong chaotic property: they have completely positive entropy. That means that the metric entropy of any non-trivial partition is positive.

In fact systems with completely positive entropy have a special name, they are called Kolmogorov. A natural question is whether every Kolmogorov system is isomorphic to a Bernoulli system. In 1973 D. Ornstein [28] gave an example of a systems which is Kolmogorov but not Bernoulli (i.e. not isomorphic to a Bernoulli shift). Later, in 1982 S. Kalikow [18] exhibited another (much more natural) example of a Kolmogorov but not Bernoulli system (see for a survey on equivalence in ergodic theory by J.P. Thouvenot [48]).

However, the first smooth example of a Kolmogorov but not Bernoulli system was given by A. Katok [20] in 1980. Moreover, it is still not known if it is possible to give a smooth example on a manifold with dimension three which is Kolmogorov but not Bernoulli. We mention also that D. Rudolph [45] had constructed a smooth example of Kolmogorov but not (loosely) Bernoulli similar to Kalikow deep (T, T^{-1}) example. A recent work of T. Austin [2] goes further and finds a continuum of distinct non-Bernoulli and Kolmogorov automorphisms.

A huge variety of natural transformations worked in the theory were proved to be Kolmogorov and most of them were proved to be Bernoulli. Y. Katznelson [21], using harmonic analysis methods proved that ergodic automorphisms of $\mathbb{T}^n$ are Bernoulli. Using a geometrical approach D. Ornstein and B. Weiss [30] proved that geodesic flows on negatively curved manifolds are Bernoulli. Later, several authors used the technique of Ornstein–Weiss to obtain the Bernoulli properties in contexts where there is a presence of some hyperbolic structure, for example, M. Ratner [38] proved the Bernoulli property for Anosov flows preserving a u-Gibbs measure and Y. Pesin [31] extended Ornstein–Weiss's argument to prove that for $C^{1+\alpha}$ nonuniformly hyperbolic diffeomorphisms, the Kolmogorov and the Bernoulli property are equivalent. In 1996 N. Chernov and C. Haskell [12] extended the equivalence of the Kolmogorov and Bernoulli property for certain nonuniformly hyperbolic maps and flows (which can possibly have singu-