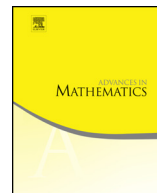




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A solution of the maximality problem for one-parameter dynamical systems

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ABSTRACT

We prove a maximality theorem for one-parameter dynamical systems that include W^* -, C^* - and multiplier one-parameter dynamical systems. Our main result is new even for one-parameter actions on commutative multiplier algebras including the algebra $C_b(\mathbb{R})$ of bounded continuous functions on $\mathbb{R}$ acted upon by translations. The methods we develop and use in our characterization of maximality include harmonic analysis, topological vector spaces and operator algebra techniques.

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1. Introduction

The current paper grew out of an effort to find a solution to the general problem of maximality of subalgebras of analytic elements associated to various dynamical systems: W^* -dynamical systems, C^* -dynamical systems and, as it will follow from the present work, multiplier dynamical systems as defined below (Section 2). The study of maximality of analytic subalgebras associated with C^* - or W^* -dynamical systems has a history of over six decades. It started with Wermer's maximality theorem [27]. Motivated by some earlier problems about approximation of continuous functions, Wermer showed that if

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$X = C(\mathbf{T})$ is the C^* -algebra of all continuous functions on $\mathbf{T} = \{t \in \mathbb{C} : |t| = 1\}$, then the norm closed subalgebra A of all functions $f \in C(\mathbf{T})$ that have an analytic extension to the unit disk $\mathbf{D} = \{s \in \mathbb{C} : |s| < 1\}$ is a maximal norm-closed subalgebra of X . Hoffman and Singer [9] and Simon [23] obtained some generalizations of Wermer's theorem for the case of compact groups with archimedean-linearly ordered Pontryagin duals.

A significant and far reaching generalization of Wermer's maximality theorem was obtained by Forelli [7] using the seminal concepts and results from his paper [6]. In [7], Forelli considered a minimal action of $\mathbb{R}$ on a locally compact Hausdorff space S , that is a homomorphism τ of $\mathbb{R}$ into the group of homeomorphisms of S onto itself such that the mapping $t \rightarrow \tau_t(s)$ is continuous for every $s \in S$ and every orbit $\{\tau_t(s) : t \in \mathbb{R}\}$, $s \in S$ is dense in S . If $f \in C_0(S)$ and $t \in \mathbb{R}$, denote $\alpha_t(f) = f \circ \tau_t$. Then α is a homeomorphism of $\mathbb{R}$ into the group of automorphisms of the C^* -algebra $X = C_0(S)$ such that the mapping $t \rightarrow \alpha_t(f)$ is continuous from $\mathbb{R}$ to $C_0(S)$ endowed with the uniform norm topology for every $f \in C_0(S)$. Forelli proved that the subalgebra $X^\alpha([0, \infty))$ of $X = C_0(S)$ consisting of all functions $f \in C_0(S)$ such that the α -spectrum of f , $sp_\alpha(f)$, as defined in the next section, is contained in $[0, \infty)$, is a maximal norm closed subalgebra of X . An equivalent description of the algebra $X^\alpha([0, \infty))$ for $X = C_0(S)$ and α as defined above is the following: $f \in X^\alpha([0, \infty))$ if and only if the mapping $t \rightarrow \alpha_t(f)(s) = (f \circ \tau_t)(s)$ has a bounded analytic extension to the upper half plane for every $s \in S$. In what follows a system (X, G, α) consisting of a C^* algebra X , a locally compact group G and a homeomorphism α from G into the group $\text{Aut}(X)$ of all automorphisms of X such that the mapping $t \rightarrow \alpha_t(x)$ is continuous from G to X with the norm topology for every $x \in X$, will be called a C^* -dynamical system. When $G = \mathbb{R}$ the system will be called a one-parameter C^* -dynamical system. If X does not contain any non trivial norm-closed α -invariant ideal, the system will be called α -simple. This is the case with Forelli system $(C_0(S), \mathbb{R}, \alpha)$ if τ is a minimal flow as defined above.

Another direction of study of maximal subalgebras is the following: If $(X, \mathbb{R}, \alpha)$ is a W^* -dynamical system consisting of a von Neumann algebra X and an action of $\mathbb{R}$ on X such that the mapping $t \rightarrow \alpha_t(x)$ is continuous from $\mathbb{R}$ to X endowed with the w^* -topology, for every $x \in X$, when is $X^\alpha([0, \infty))$ a maximal w^* -closed subalgebra of X ? In [21] Sarason noticed that Hoffman and Singer [10] basically proved the first result in this direction: $H^\infty(\mathbf{T})$ is a maximal w^* -closed subalgebra of $L^\infty(\mathbf{T})$, where $H^\infty(\mathbf{T}) \subset L^\infty(\mathbf{T})$ is the subalgebra of all $f \in L^\infty(\mathbf{T})$ that have an analytic extension to the unit disk $\mathbf{D}$. Further in [14] Muhly showed that if $(X, \mathbb{R}, \alpha)$ is an ergodic W^* -dynamical system with X an abelian von Neumann algebra, then the analytic subalgebra, $X^\alpha([0, \infty))$ as defined below is a maximal w^* -closed subalgebra of X .

Some extensions of Wermer's result to the more general case of C^* - and W^* -dynamical systems were obtained in [13] for W^* -crossed products and in [18] for C^* -crossed products. In [24] Solel has found necessary and sufficient conditions for maximality of subalgebras of analytic elements associated with periodic σ -finite W^* -dynamical systems and, further, in [25] he has considered the maximality for the particular case of an α -simple one-parameter σ -finite W^* -dynamical system, i.e. the case when X has no non-trivial

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