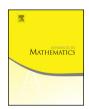


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# Singularities of mean convex level set flow in general ambient manifolds



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#### ABSTRACT

We prove two new estimates for the level set flow of mean convex domains in Riemannian manifolds. Our estimates give control – exponential in time – for the infimum of the mean curvature, and the ratio between the norm of the second fundamental form and the mean curvature. In particular, the estimates remove a stumbling block that has been left after the work of White [16,17,20], and Haslhofer–Kleiner [9], and thus allow us to extend the structure theory for mean convex level set flow to general ambient manifolds of arbitrary dimension. © 2018 Elsevier Inc. All rights reserved.

#### 1. Introduction

Let N be a Riemannian manifold. For any *compact* mean convex domain  $K_0 \subset N$  with smooth boundary, we consider the level set flow  $\{K_t\}_{t\geq 0}$  starting at  $K_0$ , i.e. the maximal family of closed sets starting at  $K_0$  that satisfies the avoidance principle when compared with any smooth mean curvature flow [6,7,11]. The level set flow of  $K_0$  coincides with the smooth mean curvature flow of  $K_0$  for as long as the latter is defined, but provides a

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canonical way to continue the evolution beyond the first singular time. Mean convexity is preserved also beyond the first singular time in the sense that  $K_{t_2} \subseteq K_{t_1}$  whenever  $t_2 \ge t_1$ .

In the last 15 years, Brian White developed a deep regularity and structure theory for mean convex level set flow [16,17,20], and recently the first author and Kleiner gave a new treatment of this theory [9]. Concerning the size of the singular set, white proved that the singular set  $S \subset N^n \times \mathbb{R}$  of any mean convex flow has parabolic Hausdorff dimension at most n-2 [16, Thm. 1.1], see also [9, Thm. 1.15]. Concerning the structure of the singular set, the main assertion one wants to prove is that all blowup limits of a mean convex flow are smooth and convex until they become extinct. In particular, one wants to conclude that all tangent flows of a mean convex flow are round shrinking spheres, round shrinking cylinders, or static planes of multiplicity one. While the theorem about the size of the singular set is known in full generality, the structure theorem has been proved up to now only under some additional assumptions [17, Thm. 1], [20, Thm. 3] and [9, Thm. 1.14]. Namely one has to restrict either to blowups at the first singular time, or to low dimensions, or to the case where the ambient manifold is Euclidean space.

As explained in [20, Appendix B], the missing step to extend the structure theorem to general ambient manifolds of arbitrary dimension is to prove that the ratio between the smallest principal curvature  $\lambda_1$  and the mean curvature H has a finite lower bound on the regular points contained in any compact subset of space—time.

The purpose of this work is to remove this stumbling block. To this end, we prove two new estimates for the level set flow of mean convex domains in Riemannian manifolds.

To state our estimates, we denote by  $\partial K_t^{\text{reg}}$  the set of regular boundary points at time t. Our first main estimate gives a lower bound for the mean curvature.

**Theorem 1.1** (Lower bound for H). There exist constants  $H_0 = H_0(K_0) > 0$  and  $\rho = \rho(K_0) < \infty$  such that

$$\inf_{\partial K_i^{reg}} H \ge H_0 e^{-\rho t}. \tag{1.2}$$

Our estimate from Theorem 1.1, as well as our second main estimate below, depends exponentially on time. It is clear from simple examples (e.g. flows in hyperbolic space), that this exponential behavior in time is the best one can possibly get.

Our second main estimate, which is the main technical achievement of this paper, controls the ratio between the norm of the second fundamental form and the mean curvature.

**Theorem 1.3** (Upper bound for |A|/H). There exist constants  $C = C(K_0) < \infty$  and  $\rho = \rho(K_0) < \infty$  such that

$$\sup_{\partial K_i^{reg}} \frac{|A|}{H} \le Ce^{\rho t}. \tag{1.4}$$

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