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Stratification of free boundary points for a two-phase variational problem ☆

Serena Dipierro [a,*], Aram L. Karakhanyan [b]

[a] *Dipartimento di Matematica "Federigo Enriques", Università di Milano, Via Cesare Saldini 50, 20133 Milano, Italy*
[b] *Maxwell Institute for Mathematical Sciences and School of Mathematics, University of Edinburgh, James Clerk Maxwell Building, Peter Guthrie Tait Road, Edinburgh EH9 3FD, United Kingdom*

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ABSTRACT

In this paper we prove the local Lipschitz regularity of the minimizers of the two-phase Bernoulli type free boundary problem arising from the minimization of the functional

$$J(u) := \int_\Omega |\nabla u|^p + \lambda_+^p \chi_{\{u>0\}} + \lambda_-^p \chi_{\{u\le 0\}}, \quad 1 < p < \infty.$$

Here $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain and $\lambda_\pm$ are positive constants such that $\lambda_+^p - \lambda_-^p > 0$. Furthermore, we show that for $p > 1$ the free boundary has locally finite perimeter and the set of non-smooth points of the free boundary is of zero $(N-1)$-dimensional Hausdorff measure. For this, our approach is new even for the classical case $p = 2$.

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* Corresponding author.
E-mail addresses: serena.dipierro@unimi.it (S. Dipierro), aram.karakhanyan@ed.ac.uk (A.L. Karakhanyan).

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Contents

1. Introduction

In this paper we study the local minimizers of

$$J(u) := \int_\Omega |\nabla u|^p + \lambda_+^p \chi_{\{u>0\}} + \lambda_-^p \chi_{\{u\le 0\}}, \quad u \in \mathcal{A}, \tag{1.1}$$

where Ω is a bounded and smooth domain in $\mathbb{R}^N$, χ_D is the characteristic function of the set $D \subset \mathbb{R}^N$, and $\lambda_\pm$ are positive constants such that

$$\Lambda := \lambda_+^p - \lambda_-^p > 0. \tag{1.2}$$

The class of admissible functions $\mathcal{A}$ is defined as follows

$$\mathcal{A} := \{u \in W^{1,p}(\Omega) : u - g \in W_0^{1,p}(\Omega), \text{ with } 1 < p < \infty\},$$

and $g \in W^{1,p}(\Omega)$ is a given boundary datum.

This type of problems arise in jet flow models with two ideal fluids, see e.g. [4] and [20] p. 126, and have been studied in [1] for $p = 2$. When the velocity $\mathbf{v}$ of the planar flow depends on the gradient of the stream function u through the power law $\mathbf{v} = |\nabla u|^{p-2}\nabla u$ (see [3]), then the resulted steady state problem admits a variational formulation with