Advances in Mathematics 328 (2018) 160–216

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Asymptotics of scalar waves on long-range asymptotically Minkowski spaces [☆]

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A R T I C L E   I N F O

Article history:
Received 11 March 2016
Accepted 10 January 2018
Available online 2 February 2018
Communicated by C. Fefferman

MSC:
primary 35L05
secondary 35P25, 58J45

Keywords:
Wave equations
Radiation field
Compound asymptotics
Compactification
Microlocal analysis
Logarithmic structure

A B S T R A C T

We show the existence of the full compound asymptotics of solutions to the scalar wave equation on long-range non-trapping Lorentzian manifolds modeled on the radial compactification of Minkowski space. In particular, we show that there is a joint asymptotic expansion at null and timelike infinity for forward solutions of the inhomogeneous equation. In two appendices we show how these results apply to certain spacetimes whose null infinity is modeled on that of the Kerr family. In these cases the leading order logarithmic term in our asymptotic expansions at null infinity is shown to be nonzero.

☆ The authors acknowledge partial support from NSF grants DMS-1500646 (DB), DMS-1361432 (AV) and DMS-1265568 (JW), and the support of NSF Postdoctoral Fellowship DMS-1103436 (DB). The authors gratefully acknowledge the hospitality of the Erwin Schrödinger Institute program "Modern Theory of Wave Equations", at which some of this work was carried out in summer 2015. The first and third author also thank the Institut Henri Poincaré for support through its "Research in Paris" program in February 2016. We also thank an anonymous referee for helpful remarks on the manuscript.
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https://doi.org/10.1016/j.aim.2018.01.012

1. Introduction

In this paper we analyze the full compound asymptotics of solutions to the scalar wave equation on long-range non-trapping Lorentzian scattering manifolds. This class of Lorentzian scattering manifolds, introduced in [1], includes short-range perturbations of the Minkowski spacetimes as well as a broad class of rather different spacetimes that admit a compactification analogous to the spherical compactification of Minkowski space. In this paper we extend these results to the more physically meaningful setting of *long-range* perturbations of gravitational type: this entails adding a term to our metric that involves a constant Bondi mass. We analyze the compound asymptotics of scalar waves near the boundary at infinity. The most interesting region for this expansion is near the boundary of the light cone, where we obtain a full understanding of the asymptotics via an appropriately scaled blow-up; the *front face* of this blow-up, i.e., the new boundary face obtained by introduction of polar coordinates, is $\mathscr{I}^+$, the null infinity of our spacetime. We analyze the Friedlander radiation field, which is given by the restriction of the rescaled solution to $\mathscr{I}^+$; in particular we find as in [1] that the asymptotics of the radiation field in the "time-delay" parameter (given by $s = 2(t - r)$ in Minkowski space and subtler here owing to long-range effects) are determined by the resonance poles of an associated Laplace-like operator for an asymptotically hyperbolic metric on the "cap" in the sphere at infinity reached by forward limits of time-like geodesics. Among the main differences of the construction here and that used for the short-range case in [1] is the necessity of a change of $\mathcal{C}^\infty$ structure on the compactified spacetime, prior to the radiation field blow-up, in order to construct the correct $\mathscr{I}^+$. We refer the reader to [1,8,12,17] for a discussion of the prior literature on radiation field decay and compound asymptotics near the light cone at infinity.

In particular, in the following theorem, the variable s is analogous to the "lapse function" $2(t - r)$ in Euclidean space; in the long-range case it is given instead by

$$s = 2(t - r) + m \log r^{-1}; \tag{1.1}$$

here the logarithmic correction has a coefficient, denoted m, related to the long-range asymptotics permitted in our spacetimes. The geometric hypotheses of the theorem are spelled out in detail in Section 3 below, and indeed we will restate the theorem in a more precise fashion in Section 8.

Theorem 1.1. *Let (M, g) be a non-trapping Lorentzian scattering manifold, and let*

$$\Box_g u = f$$

with $u \in \mathcal{C}^{-\infty}(M)$, $f \in \dot{\mathcal{C}}^\infty(M)$. Assume that u is a forward solution. Then u has a joint polyhomogeneous asymptotic expansion in $s \to \infty$, $r \to \infty$ (where r and s are as in equation (1.1))