Advances in Mathematics 328 (2018) 217–247

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Heat flow for Dirichlet-to-Neumann operator with critical growth ☆

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A R T I C L E   I N F O

Article history:
Received 25 February 2016
Accepted 10 January 2018
Available online xxxx
Communicated by the Managing Editors

MSC:
35K55
35B33
35B40

Keywords:
Dirichlet-to-Neumann operator
Critical growth
Blowup
Global solutions
Concentration phenomenon

A B S T R A C T

In this article, we study the heat flow equation for Dirichlet-to-Neumann operator with critical growth. By assuming that the initial value is lower-energy, we obtain the existence, blowup and regularity. On the other hand, a concentration phenomenon of the solution when the time goes to infinity is proved.

☆ This work was supported by the NSFC (No. 11626038, 11271305, 11531010).

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https://doi.org/10.1016/j.aim.2018.01.010

1. Introduction

Let Ω be a bounded smooth domain in $\mathbb{R}^N$, $N \geq 3$, with smooth boundary $\partial\Omega$. We denote a physical body by Ω. The electrical conductivity of Ω is represented by a bounded and positive function $\gamma(x)$. In the absence of sinks or sources of current, the equation for the potential is given by $\nabla \cdot (\gamma \nabla u) = 0$, since, by Ohm's law, $\gamma \nabla u$ represents the current flux. Given a potential function f on the boundary the induced potential $u \in H^1(\Omega)$ solves the Dirichlet problem

$$\begin{cases} \nabla \cdot (\gamma \nabla u) = 0, & \text{in } \Omega, \\ u = f, & \text{on } \partial\Omega. \end{cases}$$

The Dirichlet-to-Neumann operator, or voltage-to-current map, is given by

$$\Lambda_\gamma(f) = \left(\gamma \frac{\partial u}{\partial \varrho} \right) \Big|_{\partial\Omega},$$

where ϱ is the unit outer normal to $\partial\Omega$. In recent years, the Dirichlet-to-Neumann operator was widely studied and fruitful results were obtained (see [1,10,17,26]). The above problem is often used as a mathematical model for electrical impedance tomography (EIT), where one measures the current through the boundary caused by a family of potential functions f and recovers $\gamma(x)$ from Dirichlet-to-Neumann operator Λ_γ. We refer to [27,28] for an extensive survey of the mathematical developments in EIT.

Our aim in this paper is somewhat different as we wish to couple heat flow to the problem. Let Ω be a bounded smooth domain in $\mathbb{R}^N$, $N \geq 2$, $2^* = \frac{2N}{N-1}$. We consider the following heat flow equation for the Dirichlet-to-Neumann operator with critical growth

$$\begin{cases} -\Delta v(x,y,t) = 0, & \text{for } \ t \in \mathbb{R}^+, (x,y) \in C, \\ v(x,y,t) = 0, & \text{for } \ t \in \mathbb{R}^+, (x,y) \in \partial_L C, \\ \dfrac{\partial v(x,0,t)}{\partial t} = -\dfrac{\partial v(x,0,t)}{\partial n} + |v|^{2^*-2} v, & \text{for } \ t \in \mathbb{R}^+, (x,0) \in \Omega \times \{0\}, \\ v(x,y,0) = v_0, & \text{for } \ (x,y) \in C, \end{cases} \tag{1.1}$$

where n is the unit outer normal to $\Omega \times \{0\}$, $C = \Omega \times (0,\infty)$, $\partial_L C := \partial\Omega \times [0,\infty)$ and

$$\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \cdots + \frac{\partial^2}{\partial x_n^2} + \frac{\partial^2}{\partial y^2}.$$

Equation (1.1) can be associated with the prescribed boundary mean curvature problem. The problem have been studied extensively in the literature, see e.g. [9,18,19]. The energy functional corresponding to (1.1) is defined by