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Derived localisation of algebras and modules [☆]C. Braun ^a, J. Chuang ^b, A. Lazarev ^{a,*}^a *Department of Mathematics and Statistics, Lancaster University, Lancaster LA1 4YF, United Kingdom*^b *Department of Mathematics, City, University of London, Northampton Square, London EC1V 0HB, United Kingdom*

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ABSTRACT

For any dg algebra A , not necessarily commutative, and a subset S in $H(A)$, the homology of A , we construct its derived localisation $L_S(A)$ together with a map $A \rightarrow L_S(A)$, well-defined in the homotopy category of dg algebras, which possesses a universal property, similar to that of the ordinary localisation, but formulated in homotopy invariant terms. Even if A is an ordinary ring, $L_S(A)$ may have non-trivial homology. Unlike the commutative case, the localisation functor does not commute, in general, with homology but instead there is a spectral sequence relating $H(L_S(A))$ and $L_S(H(A))$; this spectral sequence collapses when, e.g. S is an Ore set or when A is a free ring.

We prove that $L_S(A)$ could also be regarded as a Bousfield localisation of A viewed as a left or right dg module over itself. Combined with the results of Dwyer–Kan on simplicial localisation, this leads to a simple and conceptual proof of the topological group completion theorem. Further applications include algebraic K -theory, cyclic and Hochschild homology, strictification of homotopy unital algebras, idempotent ideals,

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the stable homology of various mapping class groups and Kontsevich's graph homology.

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Contents

1.	Introduction	556
1.1.	Notation and conventions	560
2.	Prerequisites on model categories of dg algebras and modules	560
2.1.	Derived free products of dg algebras	562
2.2.	Bigraded resolutions of algebras	566
2.3.	Derived endomorphism dg algebras of bimodules	567
3.	Derived localisation of dg algebras	569
3.1.	Non-derived localisation	571
3.2.	Homotopy coherence	572
3.3.	An explicit model for localisation	573
3.4.	Homology of localisations	575
4.	Derived localisation of modules	576
4.1.	Localisations of free algebras	581
4.2.	Comparison of localisations	584
4.3.	Derived matrix localisation	585
5.	Computing localisations	588
5.1.	Localisation of degree zero homology	590
5.2.	Ore localisation	591
6.	Hochschild homology and cohomology	596
7.	Torsion modules and the localisation exact sequence	598
8.	Strictification of homotopy unital dg algebras	600
9.	Idempotent ideals and derived quotients	601
9.1.	Drinfeld's quotient	603
10.	The group completion theorem	604
10.1.	Simplicial localisation	604
10.2.	Group completion	605
11.	Localisation of dg bialgebras	607
11.1.	Cyclic homology	608
11.2.	Graph homology	610
11.3.	Algebraic K -theory	611
12.	The stable mapping class group	612
12.1.	The pair of pants gluing	612
12.2.	Punctures	613
12.3.	Open gluing	615
12.4.	Closed gluing	618
12.5.	Localisation of the dg algebra of ribbon graphs	619
	References	620

1. Introduction

Localisation of a commutative ring is among the fundamental tools in commutative algebra and algebraic geometry; it has been well-understood and documented for a long time. Let us recall the basic construction.

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