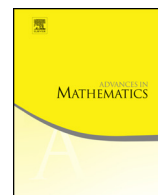




Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



Degenerating Hermitian metrics and spectral geometry of the canonical bundle

Francesco Bei

Institut Camille Jordan, Université Lyon 1, France

ARTICLE INFO

Article history:

Received 29 July 2016

Accepted 22 January 2018

Available online 12 February 2018

Communicated by the Managing Editors

MSC:

32W05

32W50

35P15

58J35

Keywords:

Hermitian complex space

Hermitian pseudometric

Canonical bundle

$\bar{\partial}$ -operator

Hodge–Kodaira Laplacian

Complex projective surface

ABSTRACT

Let (X, h) be a compact and irreducible Hermitian complex space of complex dimension m . In this paper we are interested in the Dolbeault operator acting on the space of L^2 sections of the canonical bundle of $\text{reg}(X)$, the regular part of X . More precisely let $\bar{\partial}_{m,0} : L^2\Omega^{m,0}(\text{reg}(X), h) \rightarrow L^2\Omega^{m,1}(\text{reg}(X), h)$ be an arbitrarily fixed closed extension of $\bar{\partial}_{m,0} : L^2\Omega^{m,0}(\text{reg}(X), h) \rightarrow L^2\Omega^{m,1}(\text{reg}(X), h)$ where the domain of the latter operator is $\Omega_c^{m,0}(\text{reg}(X))$. We establish various properties such as closed range of $\bar{\partial}_{m,0}$, compactness of the inclusion $\mathcal{D}(\bar{\partial}_{m,0}) \hookrightarrow L^2\Omega^{m,0}(\text{reg}(X), h)$ where $\mathcal{D}(\bar{\partial}_{m,0})$, the domain of $\bar{\partial}_{m,0}$, is endowed with the corresponding graph norm, and discreteness of the spectrum of the associated Hodge–Kodaira Laplacian $\bar{\partial}_{m,0}^* \circ \bar{\partial}_{m,0}$ with an estimate for the growth of its eigenvalues. Several corollaries such as trace class property for the heat operator associated to $\bar{\partial}_{m,0}^* \circ \bar{\partial}_{m,0}$, with an estimate for its trace, are derived. Finally in the last part we provide several applications to the Hodge–Kodaira Laplacian in the setting of both compact irreducible Hermitian complex spaces with isolated singularities and complex projective surfaces.

© 2018 Elsevier Inc. All rights reserved.

E-mail addresses: bei@math.univ-lyon1.fr, francescobei27@gmail.com.

<https://doi.org/10.1016/j.aim.2018.01.021>

0001-8708/© 2018 Elsevier Inc. All rights reserved.

Contents

0.	Introduction	751
1.	Background material	757
2.	Some abstract results	763
3.	Parabolic open subsets and Hodge–Dolbeault operator	767
4.	Main theorems	771
5.	Applications	779
5.1.	Hermitian complex spaces	779
5.2.	Self-adjoint extensions with discrete spectrum in the setting of isolated singularities	783
5.3.	The Hodge–Kodaira Laplacian on complex projective surfaces	786
	Acknowledgments	799
	References	799

0. Introduction

Consider a complex projective variety $V \subset \mathbb{CP}^n$. The regular part of V , $\text{reg}(V)$, comes equipped with a natural Kähler metric g , which is the one induced by the Fubini–Study metric of $\mathbb{CP}^n$. In particular, whenever V has a nonempty singular set, we get an incomplete Kähler manifold of finite volume. In the seminal papers [10] and [22], given a singular projective variety V , many questions with a rich interaction of topology and analysis, for instance intersection cohomology, L^2 -cohomology and Hodge theory, have been raised for the incomplete Kähler manifold $(\text{reg}(V), g)$. Some of the most important among them are the Cheeger–Goresky–MacPherson’s conjecture and the MacPherson’s conjecture. The former, which is still open, says that the maximal L^2 -de Rham cohomology groups of $(\text{reg}(V), g)$ are isomorphic to the middle perversity intersection cohomology groups of V while the latter, proved in [29], asks whether the L^2 - $\bar{\partial}$ -cohomology groups in bidegree $(0, q)$ of $(\text{reg}(V), g)$ are isomorphic to the $(0, q)$ -Dolbeault cohomology groups of $\tilde{V}$, where $\tilde{V}$ is a resolution of V à la Hironaka. Related to these problems there are many other interesting and deep analytic questions. We can mention for instance the L^2 -Stokes theorem which asks whether the maximal and minimal extension of the de Rham differential d are the same, the existence of a L^2 -Hodge decomposition for the L^2 -de Rham cohomology of $(\text{reg}(V), g)$ in terms of the L^2 - $\bar{\partial}$ -cohomology of $(\text{reg}(V), g)$, the existence of self-adjoint extensions of the Hodge–de Rham operator $d + d^t$ and the Hodge–Dolbeault operator $\bar{\partial} + \bar{\partial}^t$ with discrete spectrum, the properties of the heat operator associated to some self-adjoint extension of the Laplacian and so on. Moreover we point out that many of these problems admit a natural extension in the more general setting of Hermitian complex spaces. Several papers, during the last thirty years, have been devoted to these questions. Without any goal of completeness we can recall here [18] and [24] which concern the Cheeger–Goresky–MacPherson’s conjecture, [9], [15], [28], [27], [29] and [34] devoted to the L^2 - $\bar{\partial}$ -cohomology, [26] and [33] concerning the $\bar{\partial}$ -operator on Hermitian complex spaces, [7], [14], [30] and [35] dealing with the L^2 -Hodge decomposition and the L^2 -Stokes theorem and finally [8], [20], [23] and [31] devoted to the heat operator.

Download English Version:

<https://daneshyari.com/en/article/8904930>

Download Persian Version:

<https://daneshyari.com/article/8904930>

[Daneshyari.com](https://daneshyari.com)