

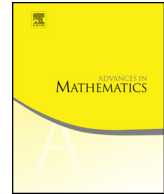


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Classification of nonnegative classical solutions to third-order equations



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ABSTRACT

In this paper, we are concerned with the third-order equations (1.1) and (1.11) with critical or subcritical nonlinearities. By applying the method of moving planes to the third-order PDEs (1.1) and (1.11), we prove that nonnegative classical solutions u to (1.1) and (1.11) are radially symmetric about some point $x_0 \in \mathbb{R}^n$ and derive the explicit forms for u in the critical case. We also prove the non-existence of nontrivial nonnegative classical solutions in the subcritical cases (see Theorem 1.1 and Theorem 1.3).

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1. Introduction

In this paper, we mainly consider the nonnegative classical solutions to the following third-order equation with critical or subcritical nonlinearities

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$$\begin{cases} (-\Delta)^{\frac{3}{2}}u(x) = u^p(x), & x \in \mathbb{R}^n, \\ u \in C_{loc}^{3,\epsilon} \cap \mathcal{L}_1(\mathbb{R}^n), & u(x) \geq 0, \quad x \in \mathbb{R}^n, \end{cases} \tag{1.1}$$

where $n \geq 4$, $1 \leq p \leq \frac{n+3}{n-3}$, $\epsilon > 0$ is arbitrarily small and the fractional Laplacian $(-\Delta)^{\frac{3}{2}} := (-\Delta)(-\Delta)^{\frac{1}{2}}$. For any $u \in C_{loc}^{1,\epsilon}(\mathbb{R}^n) \cap \mathcal{L}_1(\mathbb{R}^n)$, the nonlocal operator $(-\Delta)^{\frac{1}{2}}$ is defined by (see [9,10,15,27,33])

$$(-\Delta)^{\frac{1}{2}}u(x) = C_n P.V. \int_{\mathbb{R}^n} \frac{u(x) - u(y)}{|x - y|^{n+1}} dy := C_n \lim_{\epsilon \rightarrow 0} \int_{|y-x| \geq \epsilon} \frac{u(x) - u(y)}{|x - y|^{n+1}} dy, \tag{1.2}$$

where the constant $C_n = (\int_{\mathbb{R}^n} \frac{1 - \cos(2\pi\zeta_1)}{|\zeta|^{n+1}} d\zeta)^{-1}$ and the function space

$$\mathcal{L}_1(\mathbb{R}^n) := \left\{ u : \mathbb{R}^n \rightarrow \mathbb{R} \mid \int_{\mathbb{R}^n} \frac{|u(x)|}{1 + |x|^{n+1}} dx < \infty \right\}. \tag{1.3}$$

The definition (1.2) of the fractional Laplacian $(-\Delta)^{\frac{1}{2}}$ can also be extended further to distributions in the space $\mathcal{L}_1(\mathbb{R}^n)$ by

$$\langle (-\Delta)^{\frac{1}{2}}u, \phi \rangle = \int_{\mathbb{R}^n} u(x)(-\Delta)^{\frac{1}{2}}\phi(x) dx, \quad \forall \phi \in C_0^\infty(\mathbb{R}^n).$$

The fractional Laplacian $(-\Delta)^{\frac{1}{2}}$ can also be defined equivalently (see [12]) by Caffarelli and Silvestre’s extension method (see [1]) for $u \in C_{loc}^{1,\epsilon}(\mathbb{R}^n) \cap \mathcal{L}_1(\mathbb{R}^n)$, that is,

$$(-\Delta)^{\frac{1}{2}}u(x) := -C_n \lim_{y \rightarrow 0^+} \frac{\partial U(x, y)}{\partial y} = -C_n \lim_{y \rightarrow 0^+} \int_{\mathbb{R}^n} \frac{|x - \xi|^2 - ny^2}{(|x - \xi|^2 + y^2)^{\frac{n+3}{2}}} u(\xi) d\xi, \tag{1.4}$$

where $U(x, y)$ is the harmonic extension of $u(x)$ in $\mathbb{R}_+^{n+1} = \{(x, y) \mid x \in \mathbb{R}^n, y \geq 0\}$. Throughout this paper, we define $(-\Delta)^{\frac{3}{2}}u := (-\Delta)(-\Delta)^{\frac{1}{2}}u$ for $u \in C_{loc}^{3,\epsilon}(\mathbb{R}^n) \cap \mathcal{L}_1(\mathbb{R}^n)$, where $(-\Delta)^{\frac{1}{2}}u$ is defined by definition (1.2) and its equivalent definition (1.4). Due to the nonlocal feature of $(-\Delta)^{\frac{1}{2}}$, we need the assumption $u \in C_{loc}^{3,\epsilon}(\mathbb{R}^n)$ with arbitrarily small $\epsilon > 0$ (merely $u \in C^3$ is not enough) to guarantee that $(-\Delta)^{\frac{1}{2}}u \in C^2(\mathbb{R}^n)$ (see [12,27]), and hence u is a classical solution to equation (1.1) in the sense that $(-\Delta)^{\frac{3}{2}}u$ is pointwise well defined and continuous in the whole \mathbb{R}^n .

The equation (1.1) is closely related to the following integral equation

$$u(x) = \int_{\mathbb{R}^n} \frac{R_{3,n}}{|x - y|^{n-3}} u^p(y) dy, \tag{1.5}$$

where the Riesz potential’s constants $R_{\alpha,n} := \frac{\Gamma(\frac{n-\alpha}{2})}{\pi^{\frac{n}{2}} 2^\alpha \Gamma(\frac{\alpha}{2})}$ for $0 < \alpha < n$ (see [28]). When $p = \frac{n+3}{n-3}$, we say the conformally invariant equation (1.1) is $\dot{H}^{\frac{3}{2}}$ -critical in the sense

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