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Composition operators with a minimal commutant ☆

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ABSTRACT

Let C_φ be a composition operator on the Hardy space H^2, induced by a linear fractional self-map φ of the unit disk. We consider the question whether the commutant of C_φ is minimal, in the sense that it reduces to the weak closure of the unital algebra generated by C_φ. We show that this happens in exactly three cases: when φ is either a non-periodic elliptic automorphism, or a parabolic non-automorphism, or a loxodromic self-map of the unit disk. Also, we consider the case of a composition operator induced by a univalent, analytic self-map φ of the unit disk that fixes the origin and that is not necessarily a linear fractional map, but in exchange its Königs's domain is bounded and strictly starlike with respect to the origin, and we show that the operator C_φ has a minimal commutant. Furthermore, we provide two examples

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of univalent, analytic self-maps φ of the unit disk such that C_φ is compact but it fails to have a minimal commutant.

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0. Introduction

Let $\mathcal{B}(H)$ stand for the algebra of all bounded linear operators on a Hilbert space H and let $A \in \mathcal{B}(H)$. Recall that the *commutant* of A is defined as the family of all operators that commute with A, that is,

$$\{A\}' := \{X \in \mathcal{B}(H) \colon AX = XA\}.$$

It is a standard fact that $\{A\}'$ is a subalgebra of $\mathcal{B}(H)$ that is closed in the weak operator topology σ. We shall denote by $\operatorname{alg}(A)$ the unital algebra generated in $\mathcal{B}(H)$ by the operator A, that is,

$$\operatorname{alg}(A) := \{p(A) \colon p \text{ is a polynomial}\}.$$

It is clear that $\overline{\operatorname{alg}(A)}^{\sigma}$ is a commutative algebra with the property that $\overline{\operatorname{alg}(A)}^{\sigma} \subseteq \{A\}'$. We say that an operator A has a *minimal commutant* provided that

$$\overline{\operatorname{alg}(A)}^{\sigma} = \{A\}'.$$

It is the purpose of this paper to investigate the last equality in the case when A is a composition operator C_φ, acting on $H^2(\mathbb{D})$, the Hardy space of the unit disk, and φ is a linear fractional transformation. We will show that the equality holds precisely when the symbol is either a non-periodic elliptic automorphism, or a parabolic non-automorphism, or a loxodromic self-map of the unit disk (see definitions below).

Recall that the Hardy space $H^2(\mathbb{D})$ is the Hilbert space of all analytic functions f on the unit disk $\mathbb{D}$ that have a Taylor series expansion around the origin of the form