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Divergence of wavelet series: A multifractal analysis

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We show the relevance of a multifractal-type analysis for pointwise convergence and divergence properties of wavelet series: Depending on the sequence space which the wavelet coefficients sequence belongs to, we obtain deterministic upper bounds for the Hausdorff dimensions of the sets of points where a given rate of divergence occurs, and we show that these bounds are generically optimal, according to several notions of genericity.

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1. Introduction

Pointwise convergence properties of Fourier series have been a major challenge in analysis, culminating in the famous Carleson–Hunt theorem. Later, one direction of research has been to estimate the "divergence rate" of partial sums at exceptional points where divergence occurs. The relevant tool to measure the size of these exceptional sets is the Hausdorff dimension, thus leading to a *multifractal analysis* of divergence: Denote by $S_n f$ the partial sums of the Fourier series of a 1 periodic function f, i.e.

$$S_n f(x) = \sum_{k=-n}^{n} c_k \, e^{2i\pi kx} \qquad \text{where} \qquad c_k = \int_0^1 f(t)\, e^{-2i\pi kt} dt,$$

and consider the sets

$$E_f^\beta = \left\{ x : \limsup_{n\to+\infty} \; n^{-\beta} |S_n f(x)| > 0 \right\}.$$

J.-M. Aubry proved that, if $f \in L^p([0,1])$, and $\beta > 0$, then $\dim(E_f^\beta) \leq 1 - \beta p$ (where $\dim(A)$ denotes the Hausdorff dimension of the set A) and he showed the optimality of this result, see [1]. This was later extended and refined by F. Bayart and Y. Heurteaux, who, in particular, showed that optimality holds for generic functions of L^p (in the sense supplied by Baire categories and prevalence), see [5,6].

Such properties were also studied in the setting of wavelet series: J.-M. Aubry obtained upper bounds on the dimensions of the sets of points where a given divergence rate of the wavelet series of an L^p function occurs, see [1]. Additionally, he showed their optimality by a specific construction in the case of the Haar wavelet. This last result was recently extended by F. Bayart and Y. Heurteaux who showed that it holds generically in L^2 (in