

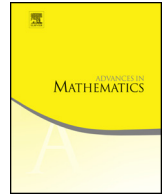


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# Monoidal categories associated with strata of flag manifolds



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## ABSTRACT

We construct a monoidal category  $\mathcal{C}_{w,v}$  which categorifies the doubly-invariant algebra  ${}^{N'(w)}\mathbb{C}[N]^{N(v)}$  associated with Weyl group elements  $w$  and  $v$ . It gives, after a localization, the coordinate algebra  $\mathbb{C}[\mathcal{R}_{w,v}]$  of the open Richardson variety associated with  $w$  and  $v$ . The category  $\mathcal{C}_{w,v}$  is realized as a subcategory of the graded module category of a quiver Hecke algebra  $R$ . When  $v = \text{id}$ ,  $\mathcal{C}_{w,v}$  is the same as the monoidal category which provides a monoidal categorification of the quantum unipotent coordinate algebra  $A_q(\mathfrak{n}(w))_{\mathbb{Z}[q, q^{-1}]}$  given by Kang–Kashiwara–Kim–Oh. We show that the category  $\mathcal{C}_{w,v}$  contains special determinantal modules  $M(w_{\leq k}\Lambda, v_{\leq k}\Lambda)$  for  $k = 1, \dots, \ell(w)$ , which commute with each other. When

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the quiver Hecke algebra  $R$  is symmetric, we find a formula of the degree of  $R$ -matrices between the determinantal modules  $M(w_{\leq k}\Lambda, v_{\leq k}\Lambda)$ . When it is of finite  $ADE$  type, we further prove that there is an equivalence of categories between  $\mathcal{C}_{w,v}$  and  $\mathcal{C}_u$  for  $w, u, v \in W$  with  $w = vu$  and  $\ell(w) = \ell(v) + \ell(u)$ .

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**Introduction**

A *quantum cluster algebra* is a non-commutative  $q$ -deformation of a *cluster algebra* which is a special  $\mathbb{Z}$ -subalgebra of a rational function field with a set of generators grouped into overlapping subsets, called *clusters*, and a procedure which makes new clusters, called *mutation*. The (quantum) cluster algebras appear naturally in various studies of mathematics including representation theory of Kac–Moody algebras. Let  $\mathfrak{g}$  be a Kac–Moody algebra of symmetric type and  $U_q(\mathfrak{g})$  its quantum group. We take an element  $w$  of the Weyl group  $W$ . It was shown in [2] that the quantum unipotent coordinate algebra  $A_q(\mathfrak{n}(w))$  associated with  $U_q(\mathfrak{g})$  and  $w$  has a quantum cluster algebra structure. It turned out that the intersection of the upper global basis and  $A_q(\mathfrak{n}(w))$  is a basis of  $A_q(\mathfrak{n}(w))$  ([15]). On the other hand, it was proved that there is a cluster algebra structure inside the coordinate ring of a *Richardson variety*. For  $w, v \in W$  with  $v \leq w$ , one can consider the *open Richardson variety*  $\mathcal{R}_{w,v}$  which is the intersection of the Schubert cell and the opposite Schubert cell attached to  $w$  and  $v$  respectively. It was shown in [17] that, when  $\mathfrak{g}$  is of finite  $ADE$  type, the coordinate algebra  $\mathbb{C}[\mathcal{R}_{w,v}]$  contains a subalgebra with the cluster algebra structure coming from a certain subcategory of module category of a preprojective algebra which is determined by  $w, v$ . The cluster variables of the initial cluster are given by the irreducible factors of the generalized minors determined by  $w_{\leq j}$

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