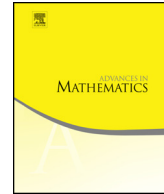




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Trilinear forms with Kloosterman fractions

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ABSTRACT

We give new bounds for $\sum_{a,m,n} \alpha_m \beta_n \nu_a e\left(\frac{am}{n}\right)$ where α_m, β_n and ν_a are arbitrary coefficients, improving upon a result of Duke, Friedlander and Iwaniec [7]. We also apply these bounds to problems on representations by determinant equations and on the equidistribution of solutions to linear equations.

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1. Introduction

For a, b, c positive integers, one defines the classical Kloosterman sum as

$$S(a, b; c) := \sum_{x \pmod{c}}^* e\left(\frac{a\bar{x} + bx}{c}\right)$$

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where, as usual, $\bar{x}$ denotes the multiplicative inverse of x modulo the denominator c , $\sum^*$ denotes a sum over the reduced residues modulo c , and $e(x) := e^{2\pi i x}$.

Several important results in number theory have been obtained by using bounds for single Kloosterman sums such as Weil's bound, $S(a, b, c) \ll (a, b, c)^{\frac{1}{2}} c^{\frac{1}{2} + \varepsilon}$, or, more recently, for averages of Kloosterman sums, in particular the bounds of Deshouillers and Iwaniec [5].

The results of [5] are particularly efficient when considering averages of $S(a, b, c)$ with weights $f(a, b, c)$ that are smooth or have at least some special structure. For many applications, however, one would like to have non-trivial bounds in the case of arbitrary weights and such bounds would be useful also when the coefficients have arithmetic/geometric nature, but have conductor which is too large to be able to employ the extra information.

In the beautiful paper [7], Duke, Friedlander and Iwaniec addressed this problem, obtaining a non-trivial bound for the following “bilinear form with Kloosterman fractions”:

$$\mathcal{B}_a(M, N) := \sum_{\substack{m \in \mathcal{M}, n \in \mathcal{N}, \\ (m, n) = 1}} \alpha_m \beta_n e\left(\frac{a\bar{m}}{n}\right),$$

where α_m, β_n are arbitrary coefficients supported on $\mathcal{M} := [M/2, M]$ and $\mathcal{N} := [N/2, N]$ respectively, and $a \neq 0$. The main result of [7] is the bound

$$\mathcal{B}_a(M, N) \ll \|\alpha\| \|\beta\| (|a| + MN)^{\frac{3}{8}} (M + N)^{\frac{11}{48} + \varepsilon}, \quad (1.1)$$

where $\|\cdot\|$ denotes the L^2 -norm. Notice that, in the important case $M \approx N$, $a \ll MN$, the bound in (1.1) saves roughly a power of $N^{\frac{1}{48}}$ over the trivial bound $\mathcal{B}_a(M, N) \ll \|\alpha\| \|\beta\| (MN)^{\frac{1}{2}}$. In this paper, we refine the arguments of Duke, Friedlander and Iwaniec and improve upon their bound, obtaining

$$\mathcal{B}_a(M, N) \ll \|\alpha\| \|\beta\| (|a| + MN)^{\frac{1}{2}} (MN)^{-\frac{3}{20} + \varepsilon} (M + N)^{\frac{1}{4}}.$$

We get a saving of $N^{\frac{1}{20}}$ when $M \approx N$, $a \ll MN$. More generally, we consider the case when an extra average over a is introduced, as this often appears in applications. We then provide a new bound for sums of the form

$$\mathcal{B}(M, N, A) := \sum_{\substack{a \in \mathcal{A}, m \in \mathcal{M}, n \in \mathcal{N}, \\ (m, n) = 1}} \alpha_m \beta_n \nu_a e\left(\vartheta \frac{a\bar{m}}{n}\right),$$

where ν_a are arbitrary coefficients supported on $\mathcal{A} := [A/2, A]$ and $\vartheta \neq 0$.

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