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On the pro-semisimple completion of the fundamental group of a smooth variety over a finite field

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A R T I C L E I N F O

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To Dima Kazhdan with gratitude and admiration

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A B S T R A C T

Let Π be the fundamental group of a smooth variety X over $\mathbb{F}_p$. Let $\overline{\mathbb{Q}}$ be an algebraic closure of $\mathbb{Q}$. Given a non-Archimedean place λ of $\overline{\mathbb{Q}}$ prime to p, consider the λ-adic pro-semisimple completion of Π as an object of the groupoid whose objects are pro-semisimple groups and whose morphisms are isomorphisms up to conjugation by elements of the neutral connected component. We prove that this object does not depend on λ. If $\dim X = 1$ we also prove a crystalline generalization of this fact.

We deduce this from the Langlands conjecture for function fields (proved by L. Lafforgue) and its crystalline analog (proved by T. Abe) using a reconstruction theorem in the spirit of Kazhdan–Larsen–Varshavsky.

We also formulate two related Conjectures E.8.1 and E.9.1. Each of them is a kind of “reciprocity law” involving a sum over all ℓ-adic cohomology theories (including the crystalline theory for $\ell = p$).

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