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The semi-infinite intersection cohomology sheaf

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To David Kazhdan, with gratitude and affection

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ABSTRACT

We introduce the semi-infinite category of sheaves on the affine Grassmannian, and construct a particular object in it, which we call the semi-infinite intersection cohomology sheaf. We relate it to several other entities naturally appearing in the geometric Langlands theory.

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0. Introduction

0.1. Our goals

0.1.1. This paper deals with sheaves on infinite-dimensional algebro-geometric objects. Specifically, we consider the *affine Grassmannian* of a reductive group G,

$$\mathrm{Gr}_G := G((t))/G[[t]],$$

and we consider sheaves that are equivariant with respect to the action of the group $N((t))$. We denote this category by $\mathrm{SI}(\mathrm{Gr}_G)$. Since $N((t))$-orbits on Gr_G are all infinite-dimensional, objects from $\mathrm{SI}(\mathrm{Gr}_G)$ necessarily have infinite-dimensional support.

We refer the reader to Sect. 0.5.7, where we explain what we mean by sheaves on infinite-dimensional objects such as Gr_G, so that the category of $N((t))$-equivariant sheaves makes sense. Let us mention, however, that in order to set up such a theory we need to work from the start with the derived category of sheaves (or, more precisely,