Advances in Mathematics 326 (2018) 54–78

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Rough bilinear singular integrals ☆

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ARTICLE INFO

Article history:
Received 1 March 2017
Accepted 12 December 2017
Available online xxxx
Communicated by C. Fefferman

MSC:
42B20
42B99

Keywords:
Singular integrals
Multilinear operators
Rough operators

ABSTRACT

We study the rough bilinear singular integral, introduced by Coifman and Meyer [8],

$$T_\Omega(f,g)(x) = \text{p.v.} \int_{\mathbb{R}^n}\int_{\mathbb{R}^n} |(y,z)|^{-2n}\Omega((y,z)/|(y,z)|)f(x-y)g(x-z)\,dydz,$$

when Ω is a function in $L^q(\mathbb{S}^{2n-1})$ with vanishing integral and $2 \le q \le \infty$. When $q=\infty$ we obtain boundedness for T_Ω from $L^{p_1}(\mathbb{R}^n)\times L^{p_2}(\mathbb{R}^n)$ to $L^p(\mathbb{R}^n)$ when $1<p_1,p_2<\infty$ and $1/p=1/p_1+1/p_2$. For $q=2$ we obtain that T_Ω is bounded from $L^2(\mathbb{R}^n)\times L^2(\mathbb{R}^n)$ to $L^1(\mathbb{R}^n)$. For q between 2 and infinity we obtain the analogous boundedness on a set of indices around the point $(1/2,1/2,1)$. To obtain our results we introduce a new bilinear technique based on tensor-type wavelet decompositions.

☆ The first author would like to acknowledge the support of the Simons Foundation (No. 315380) and of the University of Missouri Research Board and Research Council. The second author was supported by NNSF of China (No. 11701583), and the Guangdong Natural Science Foundation (No. 2017A030310054). The third author was supported by the ERC CZ grant LL1203 of the Czech Ministry of Education.

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https://doi.org/10.1016/j.aim.2017.12.013
0001-8708/

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1. Introduction

Singular integral theory was initiated in the seminal work of Calderón and Zygmund [3]. The study of boundedness of rough singular integrals of convolution type has been an active area of research since the middle of the twentieth century. Calderón and Zygmund [4] first studied the rough singular integral

$$L_\Omega(f)(x) = \text{p.v.} \int_{\mathbb{R}^n} \frac{\Omega(y/|y|)}{|y|^n} f(x-y)\, dy$$

where Ω is in $L \log L(\mathbb{S}^{n-1})$ with mean value zero and showed that L_Ω is bounded on $L^p(\mathbb{R}^n)$ for $1 < p < \infty$. The same conclusion under the less restrictive condition that Ω lies in $H^1(\mathbb{S}^{n-1})$ was obtained by Coifman and Weiss [10] and Connett [11]. The weak type $(1,1)$ boundedness of L_Ω when $n = 2$ was established by Christ and Rubio de Francia [7] and independently by Hofmann [20], both inspired by Christ's work [6]. Additionally, in unpublished work, Christ and Rubio de Francia extended this result to all dimensions $n \le 7$. The weak type $(1,1)$ property of L_Ω was proved by Seeger [28] in all dimensions and was later extended by Tao [30] to situations in which there is no Fourier transform structure. Several questions remain concerning the endpoint behavior of L_Ω, such as if the condition $\Omega \in L \log L(\mathbb{S}^{n-1})$ can be relaxed to $\Omega \in H^1(\mathbb{S}^{n-1})$, or merely $\Omega \in L^1(\mathbb{S}^{n-1})$ when Ω is an odd function. On the former there is a partial result of Stefanov [29] but not much is still known about the latter.

The bilinear counterpart of the rough singular integral linear theory is notably more intricate. To fix notation, we fix $1 < q \le \infty$ and we let Ω in $L^q(\mathbb{S}^{2n-1})$ with $\int_{\mathbb{S}^{2n-1}} \Omega \, d\sigma = 0$, where $\mathbb{S}^{2n-1}$ is the unit sphere in $\mathbb{R}^{2n}$. Coifman and Meyer [8] introduced the bilinear singular integral operator associated with Ω by

$$T_\Omega(f,g)(x) = \text{p.v.} \int_{\mathbb{R}^n} \int_{\mathbb{R}^n} K(x-y, x-z) f(y) g(z)\, dy dz, \tag{1}$$

where f, g are functions in the Schwartz class $\mathcal{S}(\mathbb{R}^n)$,

$$K(y,z) = \Omega((y,z)')/|(y,z)|^{2n},$$