

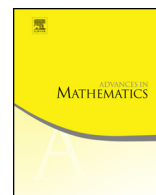


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# Gap-labelling conjecture with nonzero magnetic field

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## ABSTRACT

Given a constant magnetic field on Euclidean space  $\mathbb{R}^p$  determined by a skew-symmetric  $(p \times p)$  matrix  $\Theta$ , and a  $\mathbb{Z}^p$ -invariant probability measure  $\mu$  on the disorder set  $\Sigma$  which is by hypothesis a Cantor set, where the action is assumed to be minimal, the corresponding Integrated Density of States of any self-adjoint operator affiliated to the twisted crossed product algebra  $C(\Sigma) \rtimes_{\sigma} \mathbb{Z}^p$ , where  $\sigma$  is the multiplier on  $\mathbb{Z}^p$  associated to  $\Theta$ , takes on values on spectral gaps in the *magnetic gap-labelling group*. The *magnetic frequency group* is defined as an explicit countable subgroup of  $\mathbb{R}$  involving Pfaffians of  $\Theta$  and its sub-matrices. We conjecture that the magnetic gap labelling group is a subgroup of the magnetic frequency group. We give evidence for the validity of our conjecture in 2D, 3D, the Jordan block diagonal case and the periodic case in all dimensions.

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## 1. Introduction

The gap-labelling theorem was originally conjectured by Bellissard [\[5\]](#) in the late 1980s. It concerns the labelling of gaps in the spectrum of a Schrödinger operator (in the absence of a magnetic field) by the elements of a subgroup of  $\mathbb{R}$  which results from pairing the  $K_0$ -group of the noncommutative analog for the Brillouin zone with the tracial state defined by the probability measure on the hull. The problem arises in a mathematical version of solid state physics in the context of aperiodic tilings. Its three proofs, discovered independently by the authors of [\[15,28,6\]](#) all concern the proof of a statement in  $K$ -theory. Earlier results include the proof of the gap-labelling conjecture in 1D [\[7\]](#), 2D [\[8,49\]](#) and in 3D [\[9\]](#). A more detailed account of the history of gap-labelling theorems can be found in [Appendix B](#).

In the presence of a non-zero constant magnetic field in Euclidean space, the gap-labelling conjecture is much trickier to state, even though it was known to be the more interesting problem in spectral theory and in condensed matter physics since the 1980s, cf. [\[4\]](#). Here, we manage to give, for the first time, a precise formulation of conjectures for the magnetic gap-labelling group in all dimensions which encompass all previously known results. More precisely, in this paper we initiate the study of the gap-labelling group in the case of the magnetic Schrödinger operator on Euclidean space  $\mathbb{R}^p$  with disorder set a Cantor set  $\Sigma$  under a non-zero magnetic field  $B = \frac{1}{2}dx^t\Theta dx$ , where  $\Theta$  is a  $(p \times p)$  skew-symmetric matrix. We believe that proving (or disproving) our conjectures would constitute an important step in the understanding of aperiodic tilings under a constant magnetic field. Given a  $\mathbb{Z}^p$ -invariant probability measure  $\mu$  on  $\Sigma$ , the corresponding Integrated Density of States of any self-adjoint operator affiliated to the twisted crossed product algebra  $C(\Sigma) \rtimes_{\sigma} \mathbb{Z}^p$  takes values on spectral gaps in an explicit

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