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Quantales and Fell bundles ☆

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ABSTRACT

We study Fell bundles on groupoids from the viewpoint of quantale theory. Given any saturated upper semicontinuous Fell bundle $\pi : E \to G$ on an étale groupoid G with G_0 locally compact Hausdorff, equipped with a suitable completion C*-algebra A of its convolution algebra, we obtain a map of involutive quantales $p : \operatorname{Max} A \to \Omega(G)$, where $\operatorname{Max} A$ consists of the closed linear subspaces of A and $\Omega(G)$ is the topology of G. We study various properties of p which mimick, to various degrees, those of open maps of topological spaces. These are closely related to properties of G, π, and A, such as G being Hausdorff, principal, or topological principal, or π being a line bundle. Under suitable conditions, which include G being Hausdorff, but without requiring saturation of the Fell bundle, A is an algebra of sections of the bundle if and only if it is the reduced C*-algebra $C^*_r(G, E)$. We also prove that $\operatorname{Max} A$ is stably Gelfand. This implies the existence of a pseudogroup $\mathcal{I}_B$ and of an étale groupoid $\mathfrak{B}$ associated canonically to any sub-C*-algebra $B \subset A$. We study a correspondence between Fell bundles and sub-C*-algebras based on these constructions, and compare it to the construction of Weyl groupoids from Cartan subalgebras.

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Contents

1. Introduction

This paper draws its motivation from well known relations between C*-algebras, groupoids, and quantales. These have been, so far, pairwise relations: one is the very prolific interplay between C*-algebras and locally compact groupoids [9,33,36], which pervades much of the modern literature on operator algebras and noncommutative geometry and, in the case of étale groupoids, has led to fruitful notions of "diagonal" for C*-algebras along with a geometric understanding of them in terms of étale groupoids, inverse semigroups, and Fell bundles [5–7,12,23,24,36,37]; another is the relation between groupoids and quantales [35,39], which in particular yields a biequivalence between the bicategories of localic étale groupoids and inverse quantal frames [41], and also a representation of étendues by inverse quantal frames [40] which is an instance of the general representation of Grothendieck toposes by Grothendieck quantales meanwhile developed