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Projections of planar Mandelbrot random measures

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ABSTRACT

Let μ be a planar Mandelbrot measure and $\pi_*\mu$ its orthogonal projection on one of the principal axes. We study the thermodynamic and geometric properties of $\pi_*\mu$. We first show that $\pi_*\mu$ is exact dimensional, with $\dim(\pi_*\mu) = \min(\dim(\mu), \dim(\nu))$, where ν is the Bernoulli product measure obtained as the expectation of $\pi_*\mu$. We also prove that $\pi_*\mu$ is absolutely continuous with respect to ν if and only if $\dim(\mu) > \dim(\nu)$. Our results provides a new proof of Dekking–Grimmett–Falconer formula for the Hausdorff and box dimension of the topological support of $\pi_*\mu$, as well as a new variational interpretation. We obtain the free energy function $\tau_{\pi_*\mu}$ of $\pi_*\mu$ on a wide subinterval $[0, q_c)$ of $\mathbb{R}_+$. For $q \in [0, 1]$, it is given by a variational formula which sometimes yields phase transitions of order larger than 1. For $q > 1$, it is given by $\min(\tau_\nu, \tau_\mu)$, which can exhibit first order phase transitions. This is in contrast with the analyticity of τ_μ over $[0, q_c)$. Also, we prove the validity of the multifractal formalism for $\pi_*\mu$ at each $\alpha \in (\tau'_{\pi_*\mu}(q_c-), \tau'_{\pi_*\mu}(0+)]$.

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Contents

1. Introduction

Mandelbrot measures are statistically self-similar measures introduced in early seventies by B. Mandelbrot in [41] as a simplified model for energy dissipation in intermittent turbulence. In $\mathbb{R}^2$, such a non-trivial random measure μ is built on $[0,1]^2$ and is characterized by $\mathbb{E}(\mu([0,1]^2)) = 1$ and the equality in law

$$\mu = \sum_{0 \le i,j \le m-1} W_{i,j}\, \mu^{(i,j)} \circ S_{i,j}^{-1}, \tag{1.1}$$

where m is an integer ≥ 2, $S_{i,j}$ are similarity maps on $\mathbb{R}^2$ defined by

$$S_{i,j}(x,y) = \left(\frac{x+i}{m}, \frac{y+j}{m}\right),$$