

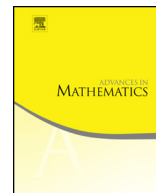


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



## Projections of planar Mandelbrot random measures

Julien Barral<sup>a,b,\*</sup>, De-Jun Feng<sup>c</sup>

<sup>a</sup> LAGA (UMR 7539), Département de Mathématiques, Université Paris 13 (Sorbonne-Paris-Cité), 99 avenue Jean-Baptiste Clément, 93430 Villetaneuse, France

<sup>b</sup> DMA (UMR 8553), Ecole Normale Supérieure, 45 rue d'Ulm, 75005 Paris, France

<sup>c</sup> Department of Mathematics, The Chinese University of Hong Kong, Shatin, Hong Kong

## ARTICLE INFO

## Article history:

Received 3 June 2016

Received in revised form 29

November 2017

Accepted 5 December 2017

Available online xxxx

Communicated by Kenneth Falconer

## MSC:

28A78

28A80

60F10

60G42

60G57

60K40

## Keywords:

Mandelbrot measures

Hausdorff dimension

Multifractals

Phase transitions

Large deviations

Branching random walk in a random environment

## ABSTRACT

Let  $\mu$  be a planar Mandelbrot measure and  $\pi_*\mu$  its orthogonal projection on one of the principal axes. We study the thermodynamic and geometric properties of  $\pi_*\mu$ . We first show that  $\pi_*\mu$  is exact dimensional, with  $\dim(\pi_*\mu) = \min(\dim(\mu), \dim(\nu))$ , where  $\nu$  is the Bernoulli product measure obtained as the expectation of  $\pi_*\mu$ . We also prove that  $\pi_*\mu$  is absolutely continuous with respect to  $\nu$  if and only if  $\dim(\mu) > \dim(\nu)$ . Our results provides a new proof of Dekking–Grimmett–Falconer formula for the Hausdorff and box dimension of the topological support of  $\pi_*\mu$ , as well as a new variational interpretation. We obtain the free energy function  $\tau_{\pi_*\mu}$  of  $\pi_*\mu$  on a wide subinterval  $[0, q_c)$  of  $\mathbb{R}_+$ . For  $q \in [0, 1]$ , it is given by a variational formula which sometimes yields phase transitions of order larger than 1. For  $q > 1$ , it is given by  $\min(\tau_\nu, \tau_\mu)$ , which can exhibit first order phase transitions. This is in contrast with the analyticity of  $\tau_\mu$  over  $[0, q_c)$ . Also, we prove the validity of the multifractal formalism for  $\pi_*\mu$  at each  $\alpha \in (\tau'_{\pi_*\mu}(q_c-), \tau'_{\pi_*\mu}(0+)]$ .

© 2017 Elsevier Inc. All rights reserved.

\* Corresponding author.

E-mail addresses: barral@math.univ-paris13.fr (J. Barral), djfeng@math.cuhk.edu.hk (D.-J. Feng).

## Contents

1.	Introduction . . . . .	641
2.	Preliminaries on multifractal formalism and Mandelbrot measures on symbolic spaces . . .	646
2.1.	Multifractal formalism on symbolic spaces . . . . .	646
2.2.	Multifractal analysis of the Mandelbrot measures on $\Sigma \times \Sigma$ . . . . .	648
3.	Main results for projections of Mandelbrot measures on the symbolic space . . . . .	650
3.1.	Absolute continuity and dimension . . . . .	651
3.2.	Validity of the multifractal formalism . . . . .	653
4.	Phase transition. Remarks and examples . . . . .	654
5.	Proofs of <a href="#">Theorem 3.1</a> , <a href="#">Theorem 3.3(1)</a> , and <a href="#">Corollary 3.5</a> . . . . .	659
5.1.	Proof of <a href="#">Theorem 3.1</a> : absolute continuity . . . . .	661
5.2.	Proof of <a href="#">Theorem 3.3(1)</a> : dimension . . . . .	664
5.3.	Proof of <a href="#">Corollary 3.5</a> : variational principle . . . . .	666
6.	Proof of <a href="#">Theorem 3.7</a> : Differentiability properties of the function $\tau$ . . . . .	669
6.1.	Differentiability over $(0, 1-]$ . . . . .	669
6.2.	Concavity of $\tau$ over $[0, 1]$ . . . . .	672
6.3.	Continuity and differentiability at 0 . . . . .	672
6.4.	The value of $\tau'(0+)$ . . . . .	673
6.5.	Differentiability at 1 . . . . .	673
6.6.	Differentiability and concavity over $(1, q_c)$ . . . . .	674
7.	Proof of <a href="#">Theorem 3.7</a> : Lower bound for the $L^q$ -spectrum . . . . .	674
8.	Proof of <a href="#">Theorem 3.7</a> : Upper bound for the $L^q$ -spectrum and validity of the multifractal formalism . . . . .	676
8.1.	Case (I) . . . . .	677
8.2.	Case (II) . . . . .	680
8.3.	Case (III) . . . . .	688
8.4.	Case (IV) . . . . .	697
9.	Positive moment estimates . . . . .	698
9.1.	Lemmas . . . . .	698
9.2.	Positive moments estimates for $X_n$ . . . . .	701
10.	Results for projections of planar Mandelbrot measures . . . . .	708
11.	Final remarks . . . . .	709
	Acknowledgments . . . . .	711
	Appendix A. Basic facts about extinction probabilities . . . . .	711
	Appendix B. Basic properties of Mandelbrot martingales in a Bernoulli environment . . . . .	713
	Appendix C. A useful lemma . . . . .	716
	References . . . . .	716

## 1. Introduction

Mandelbrot measures are statistically self-similar measures introduced in early seventies by B. Mandelbrot in [\[41\]](#) as a simplified model for energy dissipation in intermittent turbulence. In  $\mathbb{R}^2$ , such a non-trivial random measure  $\mu$  is built on  $[0, 1]^2$  and is characterized by  $\mathbb{E}(\mu([0, 1]^2)) = 1$  and the equality in law

$$\mu = \sum_{0 \leq i, j \leq m-1} W_{i,j} \mu^{(i,j)} \circ S_{i,j}^{-1}, \quad (1.1)$$

where  $m$  is an integer  $\geq 2$ ,  $S_{i,j}$  are similarity maps on  $\mathbb{R}^2$  defined by

$$S_{i,j}(x, y) = \left( \frac{x+i}{m}, \frac{y+j}{m} \right),$$

Download English Version:

<https://daneshyari.com/en/article/8905038>

Download Persian Version:

<https://daneshyari.com/article/8905038>

[Daneshyari.com](https://daneshyari.com)