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Long term regularity of the one-fluid Euler–Maxwell system in 3D with vorticity [☆]

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ABSTRACT

A basic model for describing plasma dynamics is given by the "one-fluid" Euler–Maxwell system, in which a compressible electron fluid interacts with its own self-consistent electromagnetic field. In this paper we prove long-term regularity of solutions of this system in 3 spatial dimensions, in the case of small initial data with nontrivial vorticity.

Our main conclusion is that the time of existence of solutions depends only on the size of the vorticity of the initial data, as long as the initial data is sufficiently close to a constant stationary solution.

Contents

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1. Introduction

A plasma is a collection of fast-moving charged particles and is one of the four fundamental states of matter. Plasmas are the most common phase of ordinary matter in the universe, both by mass and by volume. Essentially, all of the visible light from space comes from stars, which are plasmas with a temperature such that they radiate strongly at visible wavelengths. Most of the ordinary (or baryonic) matter in the universe, however, is found in the intergalactic medium, which is also a plasma, but much hotter, so that it radiates primarily as X-rays. We refer to [3,6] for physics references in book form.

One of the basic models for describing plasma dynamics is the Euler–Maxwell "two-fluid" model, in which two compressible ion and electron fluids interact with their own self-consistent electromagnetic field. In this paper we consider a slightly simplified version, the so-called one-fluid Euler–Maxwell system (EM) for electrons, which accounts for the interaction of electrons and the electromagnetic field, but neglects the dynamics of the ion fluid. The model describes the dynamical evolution of the functions $n_e : \mathbb{R}^3 \to \mathbb{R}$ (the density of the fluid), $v_e : \mathbb{R}^3 \to \mathbb{R}^3$ (the velocity field of the fluid), and $E', B' : \mathbb{R}^3 \to \mathbb{R}^3$ (the electric and magnetic fields), which evolve according to the coupled nonlinear system

$$\begin{cases} \partial_t n_e + \mathrm{div}(n_e v_e) = 0, \\ m_e(\partial_t v_e + v_e \cdot \nabla v_e) = -P_e \nabla n_e - e\left[E' + (v_e/c) \times B'\right], \\ \partial_t E' - c\nabla \times B' = 4\pi e n_e v_e, \\ \partial_t B' + c\nabla \times E' = 0, \end{cases} \tag{1.1}$$

together with the constrains

$$\mathrm{div}(B') = 0, \quad \mathrm{div}(E') = -4\pi e(n_e - n^0). \tag{1.2}$$

The constraints (1.2) are propagated by the flow if they are satisfied at the initial time.

There are several physical constants in the above system: $-e < 0$ is the electron's charge, m_e is the electron's mass, c denotes the speed of light, and P_e is related to the effective electron temperature (that is $k_B T_e = n^0 P_e$, where k_B is the Boltzmann constant). In the system above we have chosen, for simplicity, the quadratic adiabatic pressure law $p_e = P_e n_e^2/2$.

The system has a family of equilibrium solutions $(n_e, v_e, E', B') = (n^0, 0, 0, 0)$, where $n^0 > 0$ is a constant. Our goal here is to investigate the long-term stability properties of these solutions.