

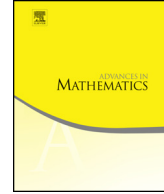


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Advances in Mathematics

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## Bressoud's conjecture



Sun Kim

Department of Mathematics, University of Illinois at Urbana-Champaign, Urbana,  
IL 61801, USA

## ARTICLE INFO

*Article history:*

Received 16 July 2017

Received in revised form 5 October  
2017

Accepted 29 November 2017

Available online 15 December 2017

Communicated by George Andrews

*MSC:*

primary 05A17

secondary 11P81, 11P84

*Keywords:*

Integer partitions

Rogers–Ramanujan identities

Rogers–Ramanujan–Gordon–

Andrews identities

Bressoud's conjecture

## ABSTRACT

In 1980, D. M. Bressoud obtained an analytic generalization of the Rogers–Ramanujan–Gordon identities. He then tried to establish a combinatorial interpretation of his identity, which specializes to many well-known Rogers–Ramanujan type identities. He proved that a certain partition identity follows from his identity in a very restrictive case and conjectured that the partition identity holds true in general. In this paper, we prove Bressoud's conjecture for the general case by providing bijective proofs.

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## 1. Introduction

The most celebrated identities in the theory of partitions are the Rogers–Ramanujan identities: for  $r = 1, 2$ ,

$$\sum_{n=0}^{\infty} \frac{q^{n^2+(r-1)n}}{(q; q)_n} = \frac{1}{(q^r, q^{5-r}; q^5)_{\infty}}, \quad (1.1)$$

*E-mail address:* sunkim2@illinois.edu.

where we follow the standard notation

$$\begin{aligned} (a; q)_0 &= 1, \\ (a; q)_n &= (1 - a)(1 - aq) \cdots (1 - aq^{n-1}), \quad n \geq 1, \\ (a; q)_\infty &= \lim_{n \rightarrow \infty} (a; q)_n, \quad |q| < 1, \\ (a_1, \dots, a_m; q)_\infty &= (a_1; q)_\infty \cdots (a_m; q)_\infty. \end{aligned}$$

The Rogers–Ramanujan identities were discovered by L. J. Rogers [15] in 1894, and S. Ramanujan [13][14, p. 330] rediscovered them. In addition, P. A. MacMahon [12] and I. J. Schur [16] independently discovered the beautiful combinatorial statement of (1.1):

For each positive integer  $n$ , and  $r = 1, 2$ , the number of partitions of  $n$  into parts greater than or equal to  $r$  with minimal difference 2 is equal to the number of partitions of  $n$  into parts congruent to  $\pm r \pmod{5}$ .

Since then, there has been considerable investigation on Rogers–Ramanujan type identities and their combinatorics. In particular, B. Gordon [7] made the first break-through by proving an infinite family of combinatorial generalizations of the Rogers–Ramanujan identities. Throughout this paper, for a partition  $\pi$ , we write the parts of  $\pi$  in weakly increasing order as follows:

$$\pi_s + \pi_{s-1} + \cdots + \pi_1, \quad \text{where } \pi_s \leq \pi_{s-1} \leq \cdots \leq \pi_1.$$

**Theorem 1.1** (*Rogers–Ramanujan–Gordon identities*). For  $1 \leq r \leq k$ , let  $A_{k,r}(n)$  be the number of partitions of  $n$  into parts  $\not\equiv 0, \pm r \pmod{2k + 1}$ . Let  $B_{k,r}(n)$  be the number of partitions  $\pi$  of  $n$  such that

$$\pi_i - \pi_{i+k-1} \geq 2,$$

and at most  $r - 1$  of the  $\pi_i$  are equal to 1. Then for all  $n \geq 0$ ,

$$A_{k,r}(n) = B_{k,r}(n). \tag{1.2}$$

Andrews [1] provided an analytic proof of Theorem 1.1, and he discovered the generating function for (1.2) in [3]:

$$\sum_{n_1, \dots, n_{k-1} \geq 0} \frac{q^{N_1^2 + N_2^2 + \cdots + N_{k-1}^2 + N_r + N_{r+1} + \cdots + N_{k-1}}}{(q; q)_{n_1} (q; q)_{n_2} \cdots (q; q)_{n_{k-1}}} = \frac{(q^r, q^{2k+1-r}, q^{2k+1}; q^{2k+1})_\infty}{(q; q)_\infty}, \tag{1.3}$$

where  $N_j = n_j + n_{j+1} + \cdots + n_{k-1}$ .

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