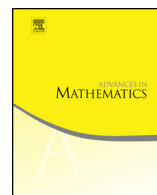




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# Heisenberg uniqueness pairs corresponding to a finite number of parallel lines

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## ABSTRACT

In this paper, we study the Heisenberg uniqueness pairs corresponding to a finite number of parallel lines  $\Gamma$ . We give a necessary condition and a sufficient condition for a subset  $\Lambda$  of  $\mathbb{R}^2$  so that  $(\Gamma, \Lambda)$  becomes a HUP.

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## 1. Introduction

Let  $\Gamma$  be a curve on  $\mathbb{R}^2$  and  $\Lambda$  be a subset of  $\mathbb{R}^2$ . We call  $(\Gamma, \Lambda)$  to be a Heisenberg uniqueness pair if

$$\hat{\mu}|_{\Lambda} = 0 \text{ implies } \mu = 0$$

for every measure  $\mu$  on  $\mathbb{R}^2$  which is supported on  $\Gamma$  and also absolutely continuous with respect to the arc length of the curve. Here  $\hat{\mu}$  stands for the Fourier transform of  $\mu$  defined by

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$$\hat{\mu}(\xi, \eta) = \int_{\mathbb{R}^2} e^{\pi i(x\xi + y\eta)} d\mu(x, y)$$

where  $\xi, \eta \in \mathbb{R}^2$ .

The terminology “Heisenberg uniqueness pair” was first introduced by Hedenmalm and Montes-Rodríguez in [5]. In that paper they also studied the cases where  $\Gamma$  are a hyperbola and two parallel straight lines. After that, Heisenberg uniqueness pairs were studied for some more well-known curves on  $\mathbb{R}^2$  by various people. For example, the case of circle and parabola were studied by Nir Lev [8] and Per Sjölin [10] respectively. Please see [2], [6], [9], [3] and [4] for more results in this direction.

In this paper we extend the results of [1] to finite number of parallel lines. However, unlike [1], we are not able to characterize Heisenberg pairs in the case  $\Gamma$  is a set of finitely many parallel lines.

### 2. Main result

In order to state our result we first set up some notations. Consider a subset  $E$  of  $\mathbb{R}$  and a point  $\xi \in E$ . Then we can define the following sets:

For  $2 \leq l \leq n$ ,  $P_l^{E,\xi} = \{\psi : E \rightarrow \mathbb{C} : \text{there is an interval } I_\xi \text{ around } \xi \text{ and functions } \varphi_j \in L^1(\mathbb{R}), j = 1, 2, \dots, l - 1 \text{ such that}$

$$\hat{\varphi}_1 + \hat{\varphi}_2\psi + \hat{\varphi}_3\psi^2 + \dots + \hat{\varphi}_{l-1}\psi^{l-2} + \psi^{l-1} = 0$$

on  $I_\xi \cap E\}$ . It is worthy to note the particular case  $l = 2$ , where  $P_2^{E,\xi} = \{\psi : E \rightarrow \mathbb{C} : \text{there is an interval } I_\xi \text{ and functions } \varphi \in L^1(\mathbb{R}), \text{ such that } \hat{\varphi} = \psi \text{ on } I_\xi \cap E\}$ . We have to work with this particular case many times throughout the paper. Note that by Wiener’s lemma ([7], page 57), if  $\psi \in P_2^{E,\xi}$ , then  $\frac{1}{\psi} \in P_2^{E,\xi}$ .

Let us consider  $n$  parallel lines  $\Gamma = \mathbb{R} \times \{\alpha_1, \alpha_2, \dots, \alpha_n\}$  where  $\alpha_{i+1} - \alpha_i = \text{constant}$ ,  $1 \leq i \leq n - 1$ . By translation and rescaling discussed in [5], it is enough to consider  $\Gamma = \mathbb{R} \times \{0, 1, 2, \dots, n - 1\}$ . If  $\mu$  is a measure such that  $\mu$  is supported on  $\Gamma$  and absolutely continuous with respect to arc length of  $\Gamma$ , then there exist functions  $f_0, f_1, \dots, f_{n-1} \in L^1(\mathbb{R})$ , such that

$$\hat{\mu}(\xi, \eta) = \hat{f}_0(\xi) + e^{\pi i\eta} \hat{f}_1(\xi) + e^{2\pi i\eta} \hat{f}_2(\xi) + \dots + e^{(n-1)\pi i\eta} \hat{f}_{n-1}(\xi). \tag{2.1}$$

Since  $\hat{\mu}$  is 2-periodic with respect to the second variable, we may assume  $\Lambda$  to be 2-periodic with respect to the second coordinate.

Let  $\Pi(\Lambda)$  be the projection of  $\Lambda$  on  $x$ -axis. That is,

$$\Pi(\Lambda) = \{\xi \in \mathbb{R} : (\xi, \eta) \in \Lambda \text{ for some } \eta \in \mathbb{R}\}.$$

Also consider the set,

$$\text{Img}(\xi) = \{\eta \in \mathbb{R} : (\xi, \eta) \in \Lambda, 0 \leq \eta < 2\}.$$

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