

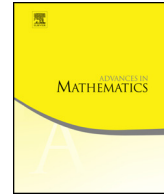


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# Sharp logarithmic estimates for positive dyadic shifts <sup>☆</sup>



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## ABSTRACT

The paper contains the study of sharp logarithmic estimates for positive dyadic shifts  $\mathcal{A}$  given on probability spaces  $(X, \mu)$  equipped with a tree-like structure. For any  $K > 0$  we determine the smallest constant  $L = L(K)$  such that

$$\int_E |\mathcal{A}f| d\mu \leq K \int_{\mathbb{R}} \Psi(|f|) d\mu + L(K) \cdot \mu(E),$$

where  $\Psi(t) = (t+1) \log(t+1) - t$ ,  $E$  is an arbitrary measurable subset of  $X$  and  $f$  is an integrable function on  $X$ . The proof exploits Bellman function method: we extract the above estimate from the existence of an appropriate special function, enjoying certain size and concavity-type conditions. As a corollary, a dual exponential bound is obtained.

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### 1. Introduction

Let  $Q \subset \mathbb{R}^d$  be a given dyadic cube and let  $\mathcal{D}(Q)$  stand for the grid of its dyadic subcubes. For a given sequence  $\alpha = (\alpha_R)_{R \in \mathcal{D}(Q)}$  of nonnegative numbers, define its Carleson constant by

$$\text{Carl}(\alpha) = \sup_{R \in \mathcal{D}(Q)} \frac{1}{|R|} \sum_{R' \in \mathcal{D}(R)} \alpha_{R'} |R'|,$$

where  $|A|$  is the Lebesgue measure of  $A$ . For any such sequence, we introduce the associated dyadic shift  $\mathcal{A}$  which acts on integrable functions  $f : Q \rightarrow \mathbb{R}$  by the formula

$$\mathcal{A}f = \sum_{R \in \mathcal{D}(Q)} \alpha_R \langle f \rangle_{R\chi_R}, \tag{1.1}$$

where  $\langle f \rangle_R = \frac{1}{|R|} \int_R f d\mu$  is the average of  $f$  over  $R$ .

The class of positive dyadic shifts arose in the works of A. Lerner during his study of the  $A_2$  theorem. Let us discuss this issue in a little more detailed manner. Assume that  $T$  is a Calderón–Zygmund operator on  $\mathbb{R}^d$  and let  $w : \mathbb{R}^d \rightarrow (0, \infty)$  be a weight satisfying Muckenhoupt’s condition  $A_2$ . The so-called  $A_2$  conjecture asked for the linear dependence of the norm  $\|T\|_{L^2(w) \rightarrow L^2(w)}$  on  $[w]_{A_2}$ , the  $A_2$  characteristic of  $w$ :

$$\|Tf\|_{L^2(w)} \leq C(T, d)[w]_{A_2} \|f\|_{L^2(w)}.$$

This question has gained a lot of interest in the recent literature (see e.g. [1,4,9,15,17–20,25]) and was finally answered in the positive by Höytönen [6], with the use of clever representation of  $T$  as an average of good dyadic shifts. Later, Lerner [11] provided a simpler proof of the  $A_2$  theorem, which avoided the use of most of the complicated techniques in [6]. The idea was to exploit a general pointwise estimate for  $T$  in terms of positive dyadic operators, proven in [10]. This allowed to reduce the  $A_2$  problem to a weighted result for the positive dyadic shifts, which had been already shown before in [8] (consult also [4] and [5]). The aforementioned pointwise bound states that for every dyadic cube  $Q$ ,

$$|Tf(x)| \lesssim \sum_{m=0}^{\infty} 2^{-\delta m} \mathcal{A}_S^m |f|(x) \quad \text{for a.e. } x \in Q, \tag{1.2}$$

where  $\delta > 0$  depends on the operator  $T$ ,  $\mathcal{S}$  is a collection of dyadic cubes which depends on  $f$ ,  $T$  and  $m$ , and  $\mathcal{A}_S^m$  are positive dyadic operators defined by

$$\mathcal{A}_S^m f(x) = \sum_{Q \in \mathcal{S}} \langle f \rangle_{Q^{(m)}} \chi_Q(x),$$

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