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Tilting modules of affine quasi-hereditary algebras

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A B S T R A C T

We discuss tilting modules of affine quasi-hereditary algebras. We present an existence theorem of indecomposable tilting modules when the algebra has a large center and use it to deduce a criterion for an exact functor between two affine highest weight categories to give an equivalence. As an application, we prove that the Arakawa–Suzuki functor (Arakawa–Suzuki, 1998) gives a fully faithful embedding of a block of the deformed BGG category of $\mathfrak{gl}_m$ into the module category of a suitable completion of degenerate affine Hecke algebra of GL_n.

1. Introduction

The notion of highest weight category and its ring theoretic counterpart, quasi-hereditary algebra introduced by Cline–Parshall–Scott [6] enables us to study representation theory of algebras of Lie theoretic origin in terms of theory of Artin algebras. Ringel's influential work [19] on tilting modules of quasi-hereditary algebras is one of the examples. In a highest weight category, there are two kinds of distinguished indecomposable modules called standard modules and costandard modules. In [19], Ringel presented tilting modules which are characterized to be filtered by both standard modules and costandard modules.

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Recently, Kleshchev [16] defined the notion of affine highest weight category and affine quasi-hereditary algebra as a graded version of the definition of Cline–Parshall–Scott [6]. Examples of affine highest weight categories include the graded module categories of Khovanov–Lauda–Rouquier (KLR) algebras of finite Lie type ([5], [13]), those of current Lie algebras etc. There are two kinds of counterparts to standard modules in an affine highest weight category called standard modules (which are of infinite length in general) and proper standard modules (which are of finite length) respectively. The counterparts to costandard modules are called proper costandard modules (which are of finite length).

In this paper, we discuss tilting modules of affine quasi-hereditary algebras, which are defined to be filtered by both standard modules and proper costandard modules. Under some conditions on the center, we prove an existence theorem of indecomposable tilting modules and deduce a simple criterion for an exact functor between two affine highest weight categories to give an equivalence.

For more detailed explanation, let H be an affine quasi-hereditary algebra over a field $\Bbbk$. By definition, H has a $\mathbb{Z}$-grading $H = \bigoplus_{n\in\mathbb{Z}} H_n$ such that we have $\dim H_n < \infty$ for any $n \in \mathbb{Z}$ and $H_n = 0$ for $n \ll 0$. Let $\{\Delta(\pi) \mid \pi \in \Pi\}$ be the set of standard modules of H, where Π is a finite set with a partial order $\leq$ parameterizing simple modules of H. For any finitely generated graded H-module V with Δ-filtration and $\pi \in \Pi$, we consider the number $(V : \Delta(\pi))$ of appearance of some grading shift of $\Delta(\pi)$ as a subquotient, which is known to be finite. We consider the following property (♠):

(♠) There is a central subalgebra $Z \subset H_{\geq 0}$ with $Z_0 = \Bbbk \cdot 1$ such that H is finitely generated as a Z-module.

Note that this is equivalent to simply saying that the algebra H is a finitely generated module over its center.

Our main results are the followings:

Theorem 1.1 *(=Theorem 3.6). Let H be an affine quasi-hereditary algebra with property (♠). Then for each $\pi \in \Pi$, there exists a unique indecomposable tilting module $T(\pi)$ such that $(T(\pi) : \Delta(\pi)) = 1$ and $(T(\pi) : \Delta(\sigma)) = 0$ for any $\sigma \not\leq \pi$. Moreover every indecomposable tilting module is isomorphic to a grading shift of $T(\pi)$ for some $\pi \in \Pi$.*

Theorem 1.2 *(=Theorem 3.9). Suppose that an exact functor Φ between the module categories over affine quasi-hereditary algebras with property (♠) induces order-preserving bijections on standard modules and on proper costandard modules. Then Φ gives an equivalence of graded categories.*

For example, the property (♠) is satisfied by KLR algebras. Therefore Theorem 1.1 says that for each affine quasi-hereditary structure of KLR algebra of finite Lie type, there exists a complete collection of indecomposable tilting modules (in the sense of Ringel).