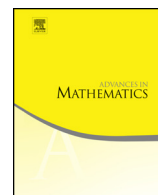




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# On the gonality and the slope of a fibered surface



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## ABSTRACT

Let  $f : X \rightarrow B$  be a locally non-trivial relatively minimal fibration of curves of genus  $g \geq 2$ . We obtain a lower bound of the slope  $\lambda(f)$  increasing with the gonality of the general fiber of  $f$ . In particular, we show that  $\lambda(f) \geq 4$  provided that  $f$  is non-hyperelliptic and  $g \geq 16$ .

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## 1. Introduction

In this paper, we always work over the complex number  $\mathbb{C}$ . A fibered surface, or simply a fibration, is a surjective proper morphism  $f : X \rightarrow B$  from a non-singular projective surface  $X$  onto a non-singular projective curve  $B$  with connected fibers. A general fiber of  $f$  is a smooth curve of genus  $g$ , which will be assumed to be at least 2 in the paper. If the general fiber of  $f$  is a hyperelliptic curve, then we call  $f$  a hyperelliptic fibration. We always assume that  $f$  is relatively minimal, i.e., there is no exceptional curve contained in the fibers of  $f$ . It is called smooth if all its fibers are smooth, isotrivial if all its smooth fibers are isomorphic to each other, and locally trivial if it is both smooth and isotrivial.

Let  $\omega_X$  (resp.  $\omega_B$ ) be the canonical sheaf of  $X$  (resp.  $B$ ), and  $\omega_{X/B} = \omega_X \otimes f^* \omega_B^\vee$  the relative canonical sheaf of  $f$ . The relative minimality of  $f$  implies that  $\omega_{X/B}$  is numerically effective, i.e.,  $\omega_{X/B} \cdot C \geq 0$  for any curve  $C \subseteq X$ . Let  $\chi(\mathcal{O}_X)$  be the Euler characteristic of the structure sheaf  $\mathcal{O}_X$ , and  $\chi_{\text{top}}(X)$  the topological Euler characteristic of  $X$ . Then we consider the following relative invariants of  $f$ :

$$\begin{cases} \omega_{X/B}^2 = \omega_X^2 - 8(g-1)(g(B)-1), \\ \deg f_* \omega_{X/B} = \chi(\mathcal{O}_X) - (g-1)(g(B)-1), \\ \delta_f = \chi_{\text{top}}(X) - 4(g-1)(g(B)-1). \end{cases}$$

They satisfy the following properties:

$$12 \deg f_* \omega_{X/B} = \omega_{X/B}^2 + \delta_f.$$

$$\delta_f \geq 0; \text{ moreover, } \delta_f = 0 \text{ iff } f \text{ is smooth.}$$

$$\deg f_* \omega_{X/B} \geq 0; \text{ moreover, } \deg f_* \omega_{X/B} = 0 \text{ iff } f \text{ is locally trivial.}$$

If  $f$  is not locally trivial, the slope of  $f$  is defined to be

$$\lambda(f) = \frac{\omega_{X/B}^2}{\deg f_* \omega_{X/B}}.$$

It follows immediately that  $0 < \lambda(f) \leq 12$ . It turns out that the slope  $\lambda(f)$  is sensible to a lot of geometric properties, both of the fibers of  $f$  and of the surface  $X$  itself (cf. [4]). We are mainly concerned about a lower bound of the slope. The main known result is the slope inequality:

$$\text{If } f \text{ is not locally trivial, then } \lambda(f) \geq \frac{4(g-1)}{g}. \quad (1.1)$$

It was first proved by Horikawa and Persson for hyperelliptic fibrations. Xiao gave a proof for general fibrations [31], and independently, Cornalba and Harris proved it for semi-stable fibrations [12] (their method was generalized to the general case later by

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