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Necessary subspace concentration conditions for the even dual Minkowski problem

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ABSTRACT

We prove tight subspace concentration inequalities for the dual curvature measures $\widetilde{C}_q(K, \cdot)$ of an n-dimensional origin-symmetric convex body for $q \geq n+1$. This supplements former results obtained in the range $q \leq n$.

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1. Introduction

Let $\mathcal{K}^n$ denote the set of convex bodies in $\mathbb{R}^n$, i.e., the family of all non-empty convex and compact subsets $K \subset \mathbb{R}^n$. The set of convex bodies having the origin as an interior point is denoted by $\mathcal{K}^n_o$ and the subset of origin-symmetric convex bodies, i.e., those sets $K \in \mathcal{K}^n_o$ satisfying $K = -K$, is denoted by $\mathcal{K}^n_e$. For $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$, let $\langle \boldsymbol{x}, \boldsymbol{y} \rangle$ denote the standard inner product and $|\boldsymbol{x}| = \sqrt{\langle \boldsymbol{x}, \boldsymbol{x} \rangle}$ the Euclidean norm. We write B_n for the n-dimensional Euclidean unit ball, i.e., $B_n = \{\boldsymbol{x} \in \mathbb{R}^n : |\boldsymbol{x}| \leq 1\}$ and $\mathbb{S}^{n-1} = \partial B_n$, where ∂K is the set of boundary points of $K \in \mathcal{K}^n$. The k-dimensional Hausdorff-measure will be denoted by $\mathcal{H}^k(\cdot)$ and instead of $\mathcal{H}^n(\cdot)$ we will also write $\mathrm{vol}(\cdot)$ for the n-dimensional volume. For a k-dimensional set $S \subset \mathbb{R}^n$ we also write $\mathrm{vol}_k(S)$ instead of $\mathcal{H}^k(S)$.

At the heart of the Brunn–Minkowski theory is the study of the volume functional with respect to the Minkowski addition of convex bodies. This leads to the theory of mixed volumes and, in particular, to the quermassintegrals $\mathrm{W}_i(K)$ of a convex body $K \in \mathcal{K}^n$. The latter may be defined via the classical Steiner formula, expressing the volume of the Minkowski sum of K and $\lambda\, B_n$, i.e., the volume of the parallel body of K at distance λ as a polynomial in λ (cf., e.g., [48, Sect. 4.2])

$$\mathrm{vol}(K + \lambda\, B_n) = \sum_{i=0}^{n} \lambda^i \binom{n}{i} \mathrm{W}_i(K). \tag{1.1}$$

A more direct geometric interpretation is given by Kubota's integral formula (cf., e.g., [48, Subsect. 5.3.2]), showing that they are — up to some constants — the means of the volumes of projections

$$\mathrm{W}_{n-i}(K) = \frac{\mathrm{vol}(B_n)}{\mathrm{vol}_i(B_i)} \int\limits_{G(n,i)} \mathrm{vol}_i(K|L)\, \mathrm{d}L, \quad i = 1, \ldots, n, \tag{1.2}$$

where integration is taken with respect to the rotation-invariant probability measure on the Grassmannian $G(n, i)$ of all i-dimensional linear subspaces and $K|L$ denotes the image of the orthogonal projection of K onto L.

A local version of the Steiner formula above leads to two important series of geometric measures, the area measures $\mathrm{S}_i(K, \cdot)$ and the curvature measures $\mathrm{C}_i(K, \cdot)$, $i = 0, \ldots, n-1$, of a convex body K. Here we will only briefly describe the area measures, since with respect to characterization problems of geometric measures they form the "primal" counterpart to the dual curvature measures we are interested in.

To this end, for $\omega \subseteq \mathbb{S}^{n-1}$ we denote by $\nu_K^{-1}(\omega) \subseteq \partial K$ the set of all boundary points of K having an outer unit normal in ω. When K is a smooth convex body, it is the inverse of the Gauss map assigning to a boundary point of K its unique outer unit normal. Moreover, for $\boldsymbol{x} \in \mathbb{R}^n$, let $r_K(\boldsymbol{x}) \in K$ be the point in K closest to $\boldsymbol{x}$. Then for a Borel set $\omega \subseteq \mathbb{S}^{n-1}$ and $\lambda > 0$ we consider the local parallel body