

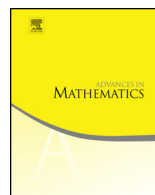


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Periodic distributions and periodic elements in modulation spaces



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ABSTRACT

We characterize periodic elements in Gevrey classes, Gelfand–Shilov distribution spaces and modulation spaces, in terms of estimates of involved Fourier coefficients, and by estimates of their short-time Fourier transforms. If $q \in [1, \infty)$, ω is a suitable weight and $(\mathcal{E}_0^E)'$ is the set of all E -periodic elements, then we prove that the dual of $M_{(\omega)}^{\infty, q} \cap (\mathcal{E}_0^E)'$ equals $M_{(1/\omega)}^{\infty, q'} \cap (\mathcal{E}_0^E)'$ by suitable extensions of Bessel's identity.

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0. Introduction

A fundamental issue in analysis concerns periodicity. For example, several problems in the theory of partial differential equations and in signal processing involve periodic

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functions and distributions. In such situations it is in general possible to discretize the problems by means of Fourier series expansions of these functions and distributions.

We recall that if f is a smooth 1-periodic function on \mathbf{R}^d , then f is equal to its Fourier series

$$\sum_{\alpha \in \mathbf{Z}^d} c(\alpha) e^{2\pi i \langle \cdot, \alpha \rangle}, \tag{0.1}$$

where the Fourier coefficients $c(\alpha)$ can be evaluated by the formula

$$c(f, \alpha) = c(\alpha) = \int_{[0,1]^d} f(x) e^{-2\pi i \langle x, \alpha \rangle} dx.$$

(Our investigations later on involve functions and distributions with more general periodics. See also [18], and Sections 1 and 2 for notations.) By the smoothness of f it follows that for every $N \geq 0$, there is a constant $C_N \geq 0$ such that

$$|c(\alpha)| \leq C_N \langle \alpha \rangle^{-N}, \tag{0.2}$$

and it follows from Weierstrass theorem that the series (0.1) is uniformly convergent (cf. e.g. [18, Section 7.2]). Here $\langle x \rangle = (1 + |x|)$.

Assume instead that f is a 1-periodic distribution on \mathbf{R}^d , and let ϕ be compactly supported and smooth on \mathbf{R}^d such that

$$\sum_{k \in \mathbf{Z}^d} \phi(\cdot - k) = 1. \tag{0.3}$$

Then f is a tempered distribution and is still equal to its Fourier series (0.1) in distribution sense. The Fourier coefficients for f are uniquely defined and can be computed by

$$c(f, \alpha) = c(\alpha) = \langle f, \phi e^{-i \langle \cdot, \alpha \rangle} \rangle, \tag{0.4}$$

and satisfy

$$|c(\alpha)| \leq C \langle \alpha \rangle^N, \tag{0.5}$$

for some constants C and N which only depend on f . In particular, $c(f, \alpha)$ is independent of the compactly supported smooth ϕ in (0.3) and (0.4). (Cf. e.g. [18, Section 7.2]. See also [32] for an early approach to formal Fourier series expansions.)

The conditions (0.2) and (0.5) are not only necessary but also sufficient for a formal Fourier series expansion (0.1) being smooth respectively a tempered distribution. Hence, by a unique extension of Parseval’s identity

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