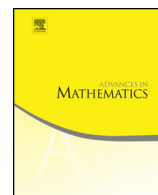




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## Canonical tilting relative generators

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## ABSTRACT

Given a relatively projective birational morphism  $f: X \rightarrow Y$  of smooth algebraic spaces with dimension of fibers bounded by 1, we construct tilting relative (over  $Y$ ) generators  $T_{X,f}$  and  $S_{X,f}$  in  $\mathcal{D}^b(X)$ . We develop a piece of general theory of strict admissible lattice filtrations in triangulated categories and show that  $\mathcal{D}^b(X)$  has such a filtration  $\mathcal{L}$  where the lattice is the set of all birational decompositions  $f: X \xrightarrow{g} Z \xrightarrow{h} Y$  with smooth  $Z$ . The  $t$ -structures related to  $T_{X,f}$  and  $S_{X,f}$  are proved to be glued via filtrations left and right dual to  $\mathcal{L}$ . We realise all such  $Z$  as the fine moduli spaces of simple quotients of  $\mathcal{O}_X$  in the heart of the  $t$ -structure for which  $S_{X,g}$  is a relative projective generator over  $Y$ . This implements the program of interpreting relevant smooth contractions of  $X$  in terms of a suitable system of  $t$ -structures on  $\mathcal{D}^b(X)$ .

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## Introduction

This paper is devoted to the categorical study of relatively projective birational morphisms  $f: X \rightarrow Y$  between smooth algebraic spaces with the dimension of fibres bounded by 1. According to a theorem of V. Danilov such a morphism has a decomposition into a sequence of blow-ups with smooth centers of codimension 2. Our goal is to find a categorical interpretation for  $f$  and for all possible intermediate contractions in terms of transformations of  $t$ -structures in the bounded derived category  $\mathcal{D}^b(X)$  of coherent sheaves on  $X$ .

Recall that T. Bridgeland, in his approach to proving the derived flop conjecture (see [9]) in dimension 3, introduced in [12] a series of  $t$ -structures in  $\mathcal{D}^b(X)$  related to a birational morphism  $f: X \rightarrow Y$  of projective varieties with fibers of dimension bounded by 1. The  $t$ -structures, with hearts  ${}^p\text{Per}(X/Y)$ , depended on an integer parameter  $p \in \mathbb{Z}$ . Under the assumption that  $f$  was a flopping contraction, he used these  $t$ -structures to define the flopped variety as a moduli space of so-called *point objects* in  ${}^{-1}\text{Per}(X/Y)$ .

In our setting of *divisorial contractions* instead of flopping contractions, we construct a system of  $t$ -structures with nice properties and interpret all possible intermediate smooth contractions between  $X$  and  $Y$  as the fine moduli spaces of simple quotients of  $\mathcal{O}_X$  in the hearts of those  $t$ -structures.

We study the partially ordered set  $\text{Dec}(f)$  of all decompositions for  $f$  into two birational morphisms with a smooth intermediate space. We prove that it is a distributive lattice and identify it with the lattice of lower ideals in a poset  $\text{Conn}(f)$ , which is a subposet in  $\text{Dec}(f)$  (see Corollary 2.15). We provide with various descriptions of  $\text{Conn}(f)$

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