

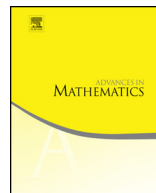


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Ramanujan coverings of graphs

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ABSTRACT

Let G be a finite connected graph, and let ρ be the spectral radius of its universal cover. For example, if G is k -regular then $\rho = 2\sqrt{k-1}$. We show that for every r , there is an r -covering (a.k.a. an r -lift) of G where all the new eigenvalues are bounded from above by ρ . It follows that a bipartite Ramanujan graph has a Ramanujan r -covering for every r . This generalizes the $r = 2$ case due to Marcus, Spielman and Srivastava [26].

Every r -covering of G corresponds to a labeling of the edges of G by elements of the symmetric group S_r . We generalize this notion to labeling the edges by elements of various groups and present a broader scenario where Ramanujan coverings are guaranteed to exist.

In particular, this shows the existence of richer families of bipartite Ramanujan graphs than was known before. Inspired by [26], a crucial component of our proof is the existence of interlacing families of polynomials for complex reflection groups. The core argument of this component is taken from [27].

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Another important ingredient of our proof is a new generalization of the matching polynomial of a graph. We define the r -th matching polynomial of G to be the average matching polynomial of all r -coverings of G . We show this polynomial shares many properties with the original matching polynomial. For example, it is real rooted with all its roots inside $[-\rho, \rho]$.

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1. Introduction

1.1. Ramanujan coverings

Throughout this paper, we assume that G is a finite, connected, undirected graph on n vertices and that A_G is its adjacency matrix. The eigenvalues of A_G are real and we denote them by

$$\lambda_n \leq \dots \leq \lambda_2 \leq \lambda_1 = \mathfrak{pf}(G),$$

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