

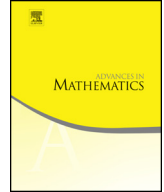


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Spectral analysis of random-to-random Markov chains [☆]



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ABSTRACT

We compute the eigenvalues and eigenspaces of random-to-random Markov chains. We use a family of maps which reveal a remarkable recursive structure of the eigenspaces, yielding an explicit and effective construction of all eigenbases starting from bases of the kernels.

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1. Introduction

Random-to-random Markov chains, also known as *random insertion* [34] and *random-to-random insertion* [36], describe the random evolution of n (ordered) objects, some of which may be identical, if someone repeatedly removes an object at random and puts it back at a random position. One can think of the objects as being books on a shelf, entries in a database, characters in a word, or cards in a deck of cards. As such, the random-to-random Markov chain constitutes a canonical card shuffling model, and can be thought of as sequentially applying a random-to-top shuffle (choose a card at random and move it to the top) and a top-to-random shuffle (move the top card to a random position) at each step.

There are several fundamental questions concerning random-to-random shuffles that have withstood analysis. This is quite striking considering that these same questions have been answered for the related random-to-top and top-to-random shuffles, which play an instrumental role in the theory of card shuffling [11] and the theory of random walks on groups and semigroups [3,5,14,23,24,31]. This paper provides answers to some of these questions and new tools to investigate others.

Our results. This paper studies random-to-random Markov chains through a spectral lens. We introduce a family of maps that allow us to compute the eigenvalues and eigenspaces of random-to-random Markov chains. Our maps reveal a remarkable recursive structure of the eigenspaces, and yield an explicit and effective construction of all eigenbases starting from bases of the kernels.

[Fig. 1](#) gives a high-level description of how our maps exploit the recursive nature of random-to-random Markov chains to construct its eigenbases. This paper introduces

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