

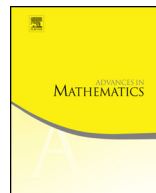


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Late-time asymptotics for the wave equation on spherically symmetric, stationary spacetimes

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ABSTRACT

We derive precise late-time asymptotics for solutions to the wave equation on spherically symmetric, stationary and asymptotically flat spacetimes including as special cases the Schwarzschild and Reissner–Nordström families of black holes. We also obtain late-time asymptotics for the time derivatives of all orders and for the radiation field along null infinity. We show that the leading-order term in the asymptotic expansion is related to the existence of the conserved Newman–Penrose quantities on null infinity. As a corollary we obtain a characterization of all solutions which satisfy Price's polynomial law τ^{-3} as a lower bound. Our analysis relies on physical space techniques and uses the vector field approach for almost-sharp decay estimates introduced in our companion paper. In the black hole case, our estimates hold in the domain of outer communications up to and including the event horizon. Our work is motivated by the

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stability problem for black hole exteriors and strong cosmic
censorship for black hole interiors.

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Contents

1.	Introduction	531
1.1.	Introduction and background	531
1.2.	Summary of main results	535
1.3.	Applications	540
1.4.	Overview of techniques	541
1.5.	The main theorems	548
1.6.	Relation with the work of Luk–Oh	554
1.7.	Outline	555
1.8.	Acknowledgments	555
2.	The geometric setting	556
2.1.	The spacetime manifold	556
2.2.	Foliations	558
2.3.	The Cauchy problem for the wave equation	562
2.4.	Assumptions for the wave equation	563
2.5.	The elliptic decomposition $\psi = \psi_0 + \psi_{\ell=1} + \psi_{\ell \geq 2}$	564
3.	The Newman–Penrose constant I_0 at null infinity	565
4.	Global decay estimates for $\psi - \int_{\mathbb{S}^2} \psi$	567
4.1.	Energy decay	567
4.2.	Pointwise decay	568
5.	Almost-sharp decay estimates for $\psi_0 = \int_{\mathbb{S}^2} \psi$ I: The case $I_0 \neq 0$	570
5.1.	Energy decay	570
5.2.	Pointwise decay	570
6.	Almost-sharp decay estimates for $\psi_0 = \int_{\mathbb{S}^2} \psi$ II: The case $I_0 = 0$	571
6.1.	Energy decay	571
6.2.	Pointwise decay	571
7.	Almost-sharp decay for the hyperboloidal derivative $Y\psi$	571
7.1.	An elliptic estimate	572
7.2.	Red-shift estimates	572
7.3.	Energy decay for $Y\psi$	573
7.4.	Global pointwise decay for $Y\psi$	579
8.	Asymptotics I: The case $I_0 \neq 0$	581
8.1.	Asymptotics of $v^2 \partial_v(r\psi)$ in the region $\mathcal{B}_\alpha$	582
8.2.	Asymptotics for the radiation field $r\psi$	584
8.3.	Global asymptotics for the scalar field ψ	591
9.	Inverting the time-translation operator T	598
9.1.	Construction of the time integral $\psi^{(1)}$	598
9.2.	The time-inverted Newman–Penrose constants $I_0^{(k)}$	602
9.3.	Initial energy norms for the time integral $\psi^{(1)}$	608
10.	Asymptotics II: The case $I_0 = 0$	609
10.1.	Asymptotics for the radiation field $r\psi$	609
10.2.	Global asymptotics for the scalar field ψ	611
Appendix A.	Energy norms	612
Appendix B.	Proof of Proposition 9.5	614
Appendix C.	Proof of Proposition 9.6	618
References		619

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