

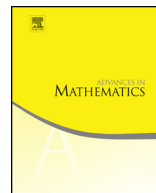


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



Analytic continuation and high energy estimates for the resolvent of the Laplacian on forms on asymptotically hyperbolic spaces



András Vasy¹

Department of Mathematics, Stanford University, CA 94305-2125, USA

ARTICLE INFO

Article history:

Received 30 July 2012

Received in revised form 12 October 2016

Accepted 31 October 2016

Available online xxxx

Communicated by the Managing Editors of AIM

MSC:

primary 58J50

secondary 35P25, 35L05, 58J47

Keywords:

Hodge Laplacian

Resolvent

High energy estimates

Analytic continuation

Asymptotically hyperbolic spaces

Asymptotically Minkowski spaces

ABSTRACT

We prove the analytic continuation of the resolvent of the Laplacian on asymptotically hyperbolic spaces on differential forms, including high energy estimates in strips. This is achieved by placing the spectral family of the Laplacian within the framework developed, and applied to scalar problems, by the author recently, roughly by extending the problem across the boundary of the compactification of the asymptotically hyperbolic space in a suitable manner. The main novelty is that the non-scalar nature of the operator is dealt with by relating it to a problem on an asymptotically Minkowski space to motivate the choice of the extension across the conformal boundary.

© 2016 Elsevier Inc. All rights reserved.

E-mail address: andras@math.stanford.edu.

¹ The author gratefully acknowledges partial support from the NSF under grant number DMS-1068742.

1. Introduction

Suppose that (X, g) is an n -dimensional asymptotically hyperbolic space with an even metric in the sense of Guillarmou [12]. That is, g is Riemannian on X , X has a compactification \overline{X} with boundary defining function x , and there is a neighborhood $U = [0, \epsilon)_x \times \partial X$ of ∂X on which g is of the warped product form $\frac{dx^2 + h}{x^2}$, with $h = h(x, \cdot)$ a smooth family of symmetric 2-cotensors on ∂X whose Taylor series at $x = 0$ is even, and $h(0, \cdot)$ is positive definite. We refer to [12] for a more geometric version, and to Graham and Lee [11, Section 5] for how to put an arbitrary asymptotically hyperbolic metric, i.e. one for which $x^2 g$ is Riemannian on \overline{X} and $|dx|_{x^2 g} = 1$ at $x = 0$, into a warped product form. We write $\overline{X}_{\text{even}}$ for \overline{X} equipped with the even smooth structure, i.e. using coordinate charts $[0, \epsilon^2)_\mu \times O$, O a coordinate chart in ∂X , in the product decomposition above, where $\mu = x^2$. (So a C^∞ function on \overline{X} is in $C^\infty(\overline{X}_{\text{even}})$ if and only if its Taylor series has only even terms at $x = 0$.)

Let Δ_k denote the Laplacian on k -forms on the complete Riemannian manifold (X, g) . Thus, Δ_k with domain $C_c^\infty(X; \Lambda^k X)$ is essentially self-adjoint, and is indeed non-negative, so in particular $(\Delta_k - \lambda)^{-1}$ exists for $\lambda \in \mathbb{C} \setminus [0, \infty)$. We show that

Theorem 1.1. *The operators*

$$\delta d(\Delta_k - \sigma^2 - (n - 2k - 1)^2/4)^{-1}, \quad d\delta(\Delta_k - \sigma^2 - (n - 2k + 1)^2/4)^{-1}$$

have a meromorphic continuation from $\text{Im } \sigma \gg 1$ to \mathbb{C} with finite rank poles and with non-trapping, resp. mildly trapping, high energy estimates in strips $|\text{Im } \sigma| < C$ if g is a non-trapping, resp. mildly trapping, metric.

Here recall that g non-trapping means that all geodesics approach ∂X as the time parameter goes to $\pm\infty$, while mildly trapping, defined in [21, Section 2], is an analytic assumption on a model problem near the trapping (roughly polynomial bounds for the model resolvent) and the nearby bicharacteristic flow; we recall this briefly at the end of Section 4. Non-trapping high-energy estimates mean that for all $C_0 > 0$ and s with $s + 3/2 > C_0$ there is $C > 0$ and $R > 0$ such that

$$\begin{aligned} \|\delta d(\Delta_k - \sigma^2 - (n - 2k - 1)^2/4)^{-1}\|_{\mathcal{L}(\mathcal{Y}_{\delta d}^{s+1}, \mathcal{X}_{\delta d}^s)} &\leq C|\sigma|, \\ \|d\delta(\Delta_k - \sigma^2 - (n - 2k + 1)^2/4)^{-1}\|_{\mathcal{L}(\mathcal{Y}_{d\delta}^{s+1}, \mathcal{X}_{d\delta}^s)} &\leq C|\sigma|, \\ |\text{Im } \sigma| < C_0, \quad |\text{Re } \sigma| > R, \end{aligned} \tag{1.1}$$

where the norms are on suitable (high-energy) Sobolev spaces, namely

$$\begin{aligned} \mathcal{X}_{\delta d}^s &= x^{-i\sigma + (n-2k-1)/2} H_{|\sigma|^{-1}}^s(\overline{X}_{\text{even}}; \Lambda^k \overline{X}_{\text{even}}), \\ \mathcal{Y}_{\delta d}^{s+1} &= x^{-i\sigma + (n-2k-1)/2+2} H_{|\sigma|^{-1}}^{s+1}(\overline{X}_{\text{even}}; \Lambda^k \overline{X}_{\text{even}}), \end{aligned}$$

Download English Version:

<https://daneshyari.com/en/article/8905180>

Download Persian Version:

<https://daneshyari.com/article/8905180>

[Daneshyari.com](https://daneshyari.com)