

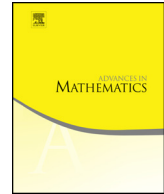


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## A duality map for quantum cluster varieties from surfaces



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### ABSTRACT

We define a canonical map from a certain space of laminations on a punctured surface into the quantized algebra of functions on a cluster variety. We show that this map satisfies a number of special properties conjectured by Fock and Goncharov. Our construction is based on the “quantum trace” map introduced by Bonahon and Wong.

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## 1. Introduction

### 1.1. Fock and Goncharov’s duality map

In their seminal paper [4], Fock and Goncharov associated, to a punctured surface  $S$ , a pair of moduli spaces denoted  $\mathcal{A}_{SL_2,S}$  and  $\mathcal{X}_{PGL_2,S}$ . These spaces are closely related to moduli spaces of  $SL_2$ - and  $PGL_2$ -local systems, respectively. Fock and Goncharov showed that certain ideas from Teichmüller theory can be understood in terms of these objects [4].

One of the main results of [4] (see also [5]) was the existence of a duality between the spaces  $\mathcal{A}_{SL_2,S}$  and  $\mathcal{X}_{PGL_2,S}$ . More precisely, Fock and Goncharov defined a space  $\mathcal{A}_{SL_2,S}(\mathbb{Z}^t)$  which is a tropicalization of  $\mathcal{A}_{SL_2,S}$ , and they constructed a canonical map

$$\mathbb{I} : \mathcal{A}_{SL_2,S}(\mathbb{Z}^t) \rightarrow \mathbb{Q}(\mathcal{X}_{PGL_2,S})$$

from this space into the algebra of rational functions on  $\mathcal{X}_{PGL_2,S}$ . The spaces  $\mathcal{A}_{SL_2,S}(\mathbb{Z}^t)$  and  $\mathcal{X}_{PGL_2,S}$  admit natural coordinates  $\{a_i\}_{i \in I}$  and  $\{X_i\}_{i \in I}$ , parametrized by the set  $I$  of edges of an ideal triangulation of  $S$ . If we number these edges so that  $I = \{1, \dots, n\}$ , then we have the following result.

**Theorem 1.1** ([4], Theorem 12.2). *The canonical functions  $\mathbb{I}(\ell)$  defined by the above construction satisfy the following properties:*

1. *For any choice of ideal triangulation,  $\mathbb{I}(\ell)$  is a Laurent polynomial in the coordinates  $X_i$  with highest term  $X_1^{a_1} \dots X_n^{a_n}$  where  $a_i$  is the coordinate of  $\ell$  associated to the edge  $i$ .*
2. *The coefficients of the Laurent polynomial  $\mathbb{I}(\ell)$  are positive integers.*
3. *For any  $\ell, \ell' \in \mathcal{A}_{SL_2,S}(\mathbb{Z}^t)$ , we have*

$$\mathbb{I}(\ell)\mathbb{I}(\ell') = \sum_{\ell'' \in \mathcal{A}_{SL_2,S}(\mathbb{Z}^t)} c(\ell, \ell'; \ell'')\mathbb{I}(\ell'')$$

where  $c(\ell, \ell'; \ell'')$  are nonnegative integers and only finitely many terms are nonzero.

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