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Existence of solutions for quasilinear random impulsive neutral differential evolution equation

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Abstract. This paper deals with the existence of solutions for quasilinear random impulsive neutral functional differential evolution equation in Banach spaces and the results are derived by using the analytic semigroup theory, fractional powers of operators and the Schauder fixed point approach. An application is provided to illustrate the theory.

Keywords: Quasilinear differential equation; Analytic semigroup; Random impulsive neutral differential equation; Fixed point theorem

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1. Introduction

In many fields of science and engineering the accurate analysis, design and assessment of systems subjected to realistic environments must take into account the potential of white noise random forces in the system properties. Randomness is acquired by a dynamical system from outside in the form of certain random action. It is this action that causes the randomness of change in the state of the system and in many other quantities determined which enables us to represent a dynamical system as a certain transformation of random inputs into random

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outputs. Randomness is intrinsic to the mathematical formulation of many phenomena, such as: fluctuations in the stock market, noise in population systems, communication networks in observed signals, etc.

The use of deterministic equations that ignore the randomness of the parameters or replace them by their mean values can result in gross errors. It is more important to consider the case when the perturbation term is rather widely impulsive in character and it is natural to expect such a situation in biological systems such as heart beats, blood flows, pulse frequency modulated systems, models for biological neural nets and automatic control problems. Therefore, perturbations of impulsive type are more realistic. This makes the study interesting.

In this paper we consider the following quasilinear random impulsive neutral differential evolution equation in a Banach space $\mathbb{X}$

$$\left.\begin{aligned} [u(t)+g(t,u(t))]' + A(t,u)u(t) &= f(t,u(t)), \quad t \in [0,T], \ t \neq \xi_k, \\ u(0) &= u_0, \\ u(\xi_k) &= b_k(\tau_k)u(\xi_k^-), \ k = 1, 2, \ldots, \end{aligned}\right\} \tag{1.1}$$

where $-A(t,u)$ is the infinitesimal generator of an analytic semigroup of operators in a Banach space $\mathbb{X}$.

Now we make the system (1.1) more precise: The function $f : J \times \mathbb{X} \to \mathbb{X}$ is uniformly bounded and continuous in all of its arguments and $u_0 \in \mathbb{X}$ and $g : J \times \mathbb{X} \to \mathbb{X}$. Take $J = [0,T]$, $T \in \mathbb{R}$ is any constant. Assume that Δ be a non-empty set and τ_k is a random variable defined from Δ to $D_k \equiv (0, d_k)$, for $k = 1, 2, \ldots$ where $0 < d_k < +\infty$. Also assume that τ_i and τ_j are independent from each other as $i \neq j$ for $i, j = 1, 2, \ldots$. Let $b_k : D_k \to \mathbb{R}$, for each $k = 1, 2, \ldots$; $\xi_0 = t_0$ and $\xi_k = \xi_{k-1} + \tau_k$ for $k = 1, 2, \ldots$; here $t_0 \in J$ is an arbitrary real number. Obviously, $t_0 = \xi_0 < \xi_1 < \cdots < \lim_{k\to\infty}\xi_k = \infty$; $u(\xi_k^-) = \lim_{t\to\xi_k} u(t)$ according to their paths with the norm $\|u\| = \sup_{0\le s\le T}|u(s)|$, for each t satisfying $0 \le t \le T$, $\|\cdot\|$ is any given norm in $\mathbb{X}$.

The problem of existence of solutions for quasilinear equation in Banach spaces has been studied by many authors. Furuya [7] and Kato [8] studied the non-homogeneous quasilinear evolution equation and the analyticity of solution in 1980's. Bahuguna [3,4] proved the existence, uniqueness and continuous dependence of a strong and local solutions to the quasilinear integrodifferential equations and also the regularity of solutions to the quasilinear equations. Oka and Tanaka [10] implemented the existence of classical solutions of abstract quasilinear integrodifferential equations. Kato [9] concentrated on the applications to PDE for the quasilinear evolution equations.

Many researchers have investigated the qualitative properties of fixed-type impulses in [15,16]. Radhakrishnan et al. [13] studied the impulsive neutral functional evolution integrodifferential systems with infinite delay. Wu et al. [17] first introduced the existence and uniqueness of solutions to random impulsive functional differential equations. Anguraj et al. [2] proved the existence and exponential stability of semilinear functional differential equations with random impulses under non-uniqueness. Yong and Wu [20] investigated the solutions of stochastic differential equations with Random impulse using Lipschitz condition. Wu et al. [19,18] discussed the boundedness and exponential stability of differential equations with random impulses.

Recently, Balachandran and Park [5] investigated the existence of solutions of quasilinear integrodifferential evolution equations by Schauder fixed point approach. Balachandran and Uchiyama [6] discussed the existence of solutions to quasilinear integrodifferential equation