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Oscillation criteria for a class of third order damped differential equations

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Abstract. The present study concerns the oscillation of a class of third-order nonlinear delay differential equations with middle term. We offer a new description of oscillation of the third-order equations in terms of oscillation of a related well studied second-order linear differential equation without damping. By using the integral averaging technique, we establish new oscillation results for this equation. Some examples are provided to illustrate the main results.

Keywords: Oscillation; Third-order; Functional delay; Differential equations

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1. INTRODUCTION

This paper is concerned with the oscillation and the asymptotic behavior of the third-order nonlinear functional differential equations with delayed argument

$$\left(r_2(\tau)\left(r_1(\tau)\left(y'(\tau)\right)^{\alpha}\right)'\right)' + \phi\left(\tau, y'(\delta(\tau))\right) + q(\tau) f(y(\sigma(\tau))) = 0, \tag{1.1}$$

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for $\tau \geq \tau_0$. Assume that the following conditions are satisfied:

(C$_1$) $r_1, r_2, q \in C\left(I, \mathbb{R}^+\right), \delta, \sigma \in C\left(I, \mathbb{R}\right)$ such that $I = [\tau_0, \infty)$, $\delta(\tau) \leq \tau$, $\sigma(\tau) \leq \tau$, $\sigma'(\tau) > 0$ and $\lim_{\tau\to\infty}\delta(\tau) = \lim_{\tau\to\infty}\sigma(\tau) = \infty$.

(C$_2$) There is a real function $p(t)$, $p(t) > 0$ such that $\phi(\tau, u) \geq k_1 p(\tau) u^\alpha$ and $\phi(\tau, -u) = -\phi(\tau, u)$.

(C$_3$) $f \in C(\mathbb{R}, \mathbb{R})$ such that $f(x)/x^\beta \geq k_2 > 0$, where α and β are quotients of odd positive integers.

A function y is called a solution of (1.1), if $y(\tau)$ satisfies (1.1) and if y, $r_1\left(y'\right)^\alpha$ and $r_2\left(r_1\left(y'\right)^\alpha\right)' \in C^1\left(\left[\tau_y, \infty\right), \mathbb{R}\right)$ for some $\tau_y \geq \tau_0$. We only consider those solutions of (1.1) which satisfy $\sup\{|y(\tau)| : \tau_1 \leq \tau < \infty\} > 0$ for any $\tau_1 \in I$ and exist on I. Such a solution is called oscillatory if it has arbitrarily large zeros, otherwise it is called nonoscillatory.

Determining oscillation and nonoscillation of functional differential equations received deal of a great interest in recent years, see the papers [1–24]. Special cases of Eq. (1.1) include the equation

$$\left(r_2(\tau)\left(r_1(\tau) y'(\tau)\right)'\right)' + p(\tau) y'(\tau) + q(\tau) f(y(g(\tau))) = 0. \tag{1.2}$$

The oscillatory behavior of solutions of (1.2) has been discussed in a number of studies, see for example the papers by Tiryaki et al. [23], Aktas et al. [3], Grace [17] and Padhi et al. [20].

In this paper, we study the oscillation and asymptotic behavior of solutions of Eq. (1.1). Our results improve and unify the results in Tiryaki et al. [23] and Elabbasy et al. [11], and to extend and generalize the earlier ones presented in Bohner et al. [7].

2. Some lemmas

In this section, we state and prove the following lemmas which we will use in the proof of our main results. For simplicity, we introduce the following notation:

$$E_0 y = y,\ E_1 y = r_1\left((E_0 y)'\right)^\alpha,\ E_2 y = r_2 (E_1 y)',\ E_3 = (E_2 y)',$$

$$R_1(\tau, \tau_1) = \int_{\tau_1}^{\tau} \frac{ds}{(r_1(s))^{\frac{1}{\alpha}}},\ R_2(\tau, \tau_1) = \int_{\tau_1}^{\tau} \frac{ds}{r_2(s)},$$

and

$$R_{12}(\tau, \tau_1) = \int_{\tau_1}^{\tau} \left(\frac{R_2(s, \tau_1)}{r_1(s)}\right)^{\frac{1}{\alpha}} ds,$$

for $\tau_0 \leq \tau_1 \leq \tau < \infty$. We suppose that

$$R_1(\tau, \tau_0) \to \infty \text{ as } \tau \to \infty$$

and

$$R_2(\tau, \tau_0) \to \infty \text{ as } \tau \to \infty.$$

Lemma 2.1. *[Grace [17]] Assume that $x(\tau)$ is a bounded solution of equation*

$$\left(r_2(\tau) x'(\tau)\right)' = G(\tau) x(h(\tau)). \tag{2.1}$$