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The stable processes on symmetric matrices

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Abstract. This paper deals with a characterization of the stable processes on the space of symmetric matrices by means of its Laplace transform. This characterization is done using a mixture with the Wishart distribution and under some independence properties. This extends the results of reference Louati et al. (2015).

Keywords: Cumulant function; Stable process; Wishart distribution

Mathematics Subject Classification: 60G51; 60G52

1. Introduction

In the last decades, Paul Lévy during his studies on the sums of the random variables had introduced the class of the stable distributions. This class of distributions has drawn a considerable interest of researchers and several works have been realized on their different aspects. For several reasons, the stable distributions are particularly important to the daily practice of the statisticians in the analysis of data belonging to many areas of application. The first reason is met when there are solid theoretical reasons for expecting a non-Gaussian stable model, *e.g.*, hitting times for a Brownian motion yielding a Lévy distribution. The second reason is dealing with the generalized central limit theorem. Because of these probabilistic important properties, many studies have been carried out and some comprehensive books have been published. We may refer to [13]. We may also mention [11] which gave a

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complete description of the basic facts related to the stable distributions and their practical applications. The stable distributions appear in several areas, such as internet traffic and communication systems (see [13]), biology (see [12]), and ecology (see [1]). In connection with these studies, and recently, several works have been proposed to discuss and to describe the stability in terms of processes. In this direction, [7] gave a characterization of the real stable process $Y = (Y(t))_{t\geq 0}$ using a mixture with an exponential standard random variable T independent of Y. Louati et al. [10] extended this result by substituting the exponential distribution by a gamma distribution and found a new characterization of the stable processes on the real line. As extension of the gamma distributions, the Wishart distributions have received a lot of attention, since their importance in relevant applications using, mainly, the graphical Gaussian modeling. The essence of graphical models in multivariate analysis is to identify independencies and conditional independencies between various groups of variables. Moreover, the Wishart distribution plays a major role in the estimation of the covariance matrices in multivariate statistics. The structure of the Wishart distribution has been studied for a long time. Nevertheless, several results about the Wishart and its derived distributions (especially, the Riesz ones) were only obtained recently (see [2–4,6,8,9] and [14]).

Since the Wishart distribution represents the natural extension of the gamma one on the cone of symmetric matrices, then in this paper, we extend the works of [7] and [10] and we characterize the class of stable processes under some independence conditions related to these processes and using the mixture with a random Wishart matrix.

The article is organized as follows: In Section 2, we recall some definitions and give some preliminary results relevant to the Wishart distribution and the multivariate stable distributions. In Section 3, we establish, under some independent conditions and using the concept of the mixture, a characterization of the multivariate stable processes on the space of symmetric matrices by their Laplace transforms.

2. Preliminaries

To make clear the results of this paper, we first recall some notations and review some characteristic properties concerning the Wishart distribution on the space of real symmetric matrices and the multivariate stable processes.

2.1. *The Wishart distribution*

Let E be the Euclidean space of (r, r) real symmetric matrices with a dimension $n = \frac{r(r+1)}{2}$ and the scalar product $\langle x, y\rangle = \mathrm{tr}(xy)$. If μ is a positive random measure on E, we denote by

$$L_\mu(\theta) = \int_E \exp(\langle\theta, x\rangle)\mu(dx) < \infty$$

its Laplace transform and

$$\Theta(\mu) = int\{\theta \in E^*;\ L_\mu(\theta) < \infty\}.$$

Let μ be a probability measure such that $\Theta(\mu) \neq \emptyset$. We define the cumulant function of the measure μ by

$$k_\mu(\theta) = \ln\left(L_\mu(\theta)\right). \tag{2.1}$$

The class of the Wishart distributions on the space E depends on two parameters p and σ. The first one is the scale parameter $p > \frac{r-1}{2}$ for which we have a convolution semi-group.

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