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# Sharp seasonal threshold property for cooperative population dynamics with concave nonlinearities

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## ABSTRACT

We consider a biological population whose environment varies periodically in time, exhibiting two very different “seasons”: one is favorable and the other one is unfavorable. For monotone differential models with concave nonlinearities, we address the following question: the system’s period being fixed, under what conditions does there exist a critical duration for the unfavorable season? By “critical duration” we mean that above some threshold, the population cannot sustain and extincts, while below this threshold, the system converges to a unique periodic and positive solution. We term this a “sharp seasonal threshold property” (SSTP, for short).

Building upon a previous result, we obtain sufficient conditions for SSTP in any dimension and apply our criterion to a two-dimensional model featuring juvenile and adult populations of insects.

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## 1. Introduction

We study differential dynamical systems arising from nonlinear periodic positive differential equations of the form

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$$\frac{dx}{dt} = F(t, x), \tag{1.1}$$

where  $F$  is monotone and concave in  $x$ . These systems exhibit well-known contraction properties when  $F$  is continuous (see [7], [9], [10]). We extend in Theorem 1 these properties to non-linearities that are only piecewise-continuous in time. This extension is motivated by the study of typical seasonal systems in population dynamics.

We denote by  $\theta \in [0, 1]$  the proportion of the year spent in unfavorable season. Then, we convene that time  $t$  belongs to an unfavorable (resp. a favorable) season if  $nT \leq t < (n + \theta)T$  (resp. if  $(n + \theta)T \leq t < (n + 1)T$ ) for some  $n \in \mathbb{Z}_+$ . In other words, we study the solutions to:

$$\frac{dX}{dt} = G(\pi_\theta(t), X), \quad \pi_\theta(t) = \begin{cases} \pi^U & \text{if } \frac{t}{T} - \lfloor \frac{t}{T} \rfloor \in [0, \theta), \\ \pi^F & \text{if } \frac{t}{T} - \lfloor \frac{t}{T} \rfloor \in [\theta, 1), \end{cases} \tag{1.2}$$

for some  $G : \mathcal{P} \times \mathbb{R}^N \rightarrow \mathbb{R}^N$ , with  $\pi^U, \pi^F \in \mathcal{P}$  where  $\mathcal{P}$  is the parameter space. We are looking for conditions ensuring that a sharp seasonal threshold property holds, that is:

$$\exists \theta_* \in [0, 1] \text{ such that } \begin{cases} \text{if } \theta < \theta_*, \exists ! q : \mathbb{R}_+ \rightarrow \mathbb{R}^N, T\text{-periodic, } q \gg 0 \text{ and} \\ \forall X_0 \in \mathbb{R}_+^N \setminus \{0\}, X \text{ converges to } q, \\ \text{if } \theta > \theta_*, \forall X_0 \in \mathbb{R}_+^N, X \text{ converges to } 0. \end{cases} \tag{SSTP}$$

Ecologically, the respective duration of dry and wet seasons is crucial for population sustainability in various species. The property (SSTP) means that if the dry season is longer than  $\theta_*T$  then the population collapses and if it is shorter than the population densities will tend to be periodic.

Assume that  $F(t, 0) \equiv 0$ . Thanks to the contraction properties of concave nonlinearities, the whole problem reduces to the study of the Floquet eigenvalue with maximum modulus of the linearization of (1.1) at  $X = 0$ :

$$\frac{dz}{dt} = D_x F(t, 0)z. \tag{1.3}$$

In fact, this eigenvalue is equal to the spectral radius of the Poincaré application for (1.3), which we compute here for piecewise-autonomous systems.

Our proof uses the Perron–Frobenius theorem and relies on the Perron eigenvalue and (left and right) eigenvectors. The importance of this eigenvalue for quantifying the effects of seasonality has been acknowledged continuously in mathematical biology in at least three application fields: circadian rhythms (in particular in connection with cell division and tumor growth), harvesting and epidemiology.

It was noted in [5] that Floquet eigenvalue with maximum modulus of (1.3) is always larger than the Perron eigenvalue of some averaged (over a period) matrix  $\overline{F}$  defined from the entries of  $D_x F(t, 0)$ . There has been a continued interest in this eigenvalue for linear

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