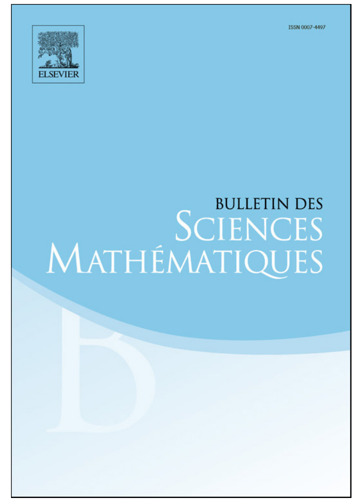
Accepted Manuscript



Non-emptiness of Brill-Noether Loci over very general quintic hypersurface

Krishanu Dan, Sarbeswar Pal

PII: S0007-4497(18)30041-1
DOI: https://doi.org/10.1016/j.bulsci.2018.05.001
Reference: BULSCI 2753

To appear in: *Bulletin des Sciences Mathématiques*

Received date: 11 October 2017

NON-EMPTINESS OF BRILL-NOETHER LOCI OVER VERY GENERAL QUINTIC HYPERSURFACE

KRISHANU DAN AND SARBESWAR PAL

Abstract. In this article we study Brill-Noether loci of moduli space of stable bundles over smooth surfaces. We define Petri map as an analogy with the case of curves. We show the non-emptiness of certain Brill-Noether loci over very general quintic hypersurface in $\mathbb{P}^3$, and use the Petri map to produce components of expected dimension.

1. Introduction

Let X be a smooth, irreducible, projective variety of dimension n over $\mathbb{C}$, H be an ample divisor on X, and let $\mathcal{M} := \mathcal{M}_{X,H}(r; c_1, \cdots, c_s)$ be the moduli space of rank r, H-stable vector bundles E over X with Chern classes $c_i(E) = c_i$, where $s := \min\{r, n\}$. A Brill-Noether locus $B^k_{r,X,H}$ is a closed subscheme of $\mathcal{M}$ whose support consists of points $E \in \mathcal{M}$ such that $h^0(X, E) \geq k + 1$. Göttsche et al ([6]) and M. He ([7]) studied the Brill-Noether loci of stable bundles over $\mathbb{P}^2$, and Yoshioka ([15], [16]), Markman ([4]), Leyenson ([10], [11]) studied them for $K3$ surfaces. In the case of smooth, projective, irreducible curves C over $\mathbb{C}$, the Brill-Noether loci of the moduli space, $\mathrm{Pic}^d(C)$, of degree d line bundles on C is well-studied. The questions like non-emptiness, connectedness, irreducibility, singular locus etc of Brill-Noether loci are known when C is a general curve in the sense of moduli (see e.g. [1]). This concept was generalized for vector bundles over curves by Newstead, Teixidor and others. For an account of the results and history in this case see [5] and the references therein.

Recently, in [2], the authors have constructed Brill-Noether loci over higher dimensional varieties under the additional cohomology vanishing assumptions: $H^i(X, E) = 0, \forall i \geq 2$ and for all $E \in \mathcal{M}$. This is a natural generalization of the Brill-Noether loci over the curves for higher dimensional varieties. In [2], [3], the authors gave several examples of non-empty Brill-Noether loci, and examples of Brill-Noether loci where "expected dimension" is not same as the exact dimension. In all these examples, the canonical bundle has no non-zero sections. In this article, we define "Petri map" over a smooth projective variety with canonical bundle ample, as an analogue of that for curves. Similar to the case of curves, the injectivity of the "Petri map" implies the existence of smooth points in Brill-Noether

2010 *Mathematics Subject Classification.* 14J60, 14H51.

Key words and phrases. Vector Bundles, Brill-Noether loci, Moduli space, Surfaces.