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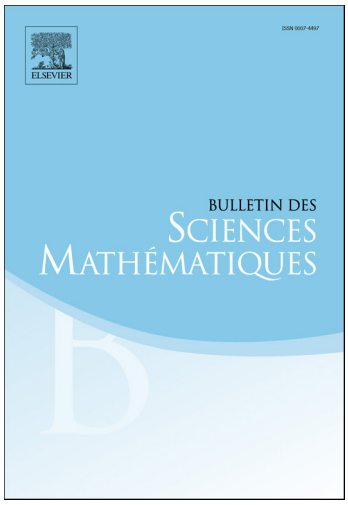

Schrödinger–Hardy systems involving two Laplacian operators in the Heisenberg group

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SCHRÖDINGER–HARDY SYSTEMS INVOLVING TWO LAPLACIAN OPERATORS IN THE HEISENBERG GROUP

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Abstract. In this paper we first prove existence of nontrivial nonnegative solutions of a Schrödinger–Hardy system in the Heisenberg group, driven by two possibly different Laplacian operators. The main originality of the paper is to work in the Heisenberg group. In fact several new theorems have to be proved in order to overcome the difficulties arising in the new framework, also due to the presence of the Hardy terms and the fact that the nonlinearities do not necessarily satisfy the Ambrosetti–Rabinowitz condition. Finally, we discuss and prove existence even for systems in the Heisenberg group, including critical nonlinear terms.

Key Words: Heisenberg group; entire solutions; Schrödinger–Hardy systems; subelliptic critical systems.

Résumé. Dans ce travail nous prouvons d'abord l'existence de solutions non négatives non triviales pour une classe de systèmes de Schrödinger–Hardy dans le groupe de Heisenberg, entraînés par deux opérateurs différents du type de Laplace. L'originalité de cet article porte surtout dans l'analyse developée dans le cadre du groupe de Heisenberg. En fait, plusieurs nouveaux théorèmes doivent être prouvés afin de surmonter les difficultés découlant du nouveau cadre, également en raison de la présence des termes de Hardy et du fait que les non–linéarités ne satisfont pas nécessairement la condition de Ambrosetti et Rabinowitz. Enfin, nous discutons et prouvons l'existence même pour les systèmes dans le groupe de Heisenberg, y compris dans le cas des termes non linéaires critiques.

Key Words: Le groupe de Heisenberg; solutions entières; Schrödinger–Hardy systèmes; systèmes critiques subelliptiques.

Mathematics Subject Classification(2010): 35R03, 35H20; 35J70; 35B33; 35A15

1. Introduction

In this paper we deal with the Schrödinger–Hardy system

$$\begin{cases} -\Delta^m_{\mathbb{H}^n} u + a(q)|u|^{m-2}u - \mu\psi^m \dfrac{|u|^{m-2}u}{r(q)^m} = H_u(q,u,v) \quad \text{in } \mathbb{H}^n, \\ -\Delta^p_{\mathbb{H}^n} v + b(q)|v|^{p-2}v - \sigma\psi^p \dfrac{|v|^{p-2}v}{r(q)^p} = H_v(q,u,v) \quad \text{in } \mathbb{H}^n, \end{cases} \tag{1.1}$$

where μ and σ are real parameters, $Q = 2n+2$ is the homogeneous dimension of the Heisenberg group $\mathbb{H}^n$, $1 < p < Q$, $1 < m \le p < m^* = mQ/(Q-m)$ and $\Delta^\wp_{\mathbb{H}^n}$ is the $\wp$–Laplacian operator on $\mathbb{H}^n$, $\wp > 1$, which is defined by

$$\Delta^\wp_{\mathbb{H}^n}\varphi = \mathrm{div}_{\mathbb{H}^n}(|D_{\mathbb{H}^n}\varphi|^{\wp-2}_{\mathbb{H}^n} D_{\mathbb{H}^n}\varphi)$$

along any $\varphi \in C_0^\infty(\mathbb{H}^n)$, that is $\Delta^\wp_{\mathbb{H}^n}$ is the familiar horizontal $\wp$–Laplacian operator.

The importance of studying problems involving (m,p) systems or equations has been attracting increasing attention on existence and qualitative properties of solutions, but the vectorial case is much harder. Indeed, (1.1) has a relevant physical interpretation in applied sciences as well as a mathematical challenge in overcoming the new difficulties intrinsic to (1.1). For general motivations