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Asymptotic behavior of Poisson integrals in a cylinder and its application to the representation of harmonic functions ☆

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Our first aim in this paper is to deal with asymptotic behavior of Poisson integrals in a cylinder. Next Carleman's formula for harmonic functions in it is also proved. As an application of them, we finally give the integral representation of harmonic functions in a cylinder.

1. Introduction and main results

Let $\mathbf{R}$ be the set of all real numbers. The boundary and the closure of a set E in n-dimensional Euclidean space $\mathbf{R}^n$ $(n \geq 2)$ are denoted by ∂E and $\overline{E}$ respectively. For positive functions h_1 and h_2, we say that $h_1 \lesssim h_2$ if $h_1 \leq ch_2$ for some constant $c > 0$. If $h_1 \lesssim h_2$ and $h_2 \lesssim h_1$, then we say that $h_1 \approx h_2$.

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Let Δ_n be the Laplace operator and Ω be a bounded domain in $\mathbf{R}^{n-1}$ with smooth boundary $\partial\Omega$. Consider the Dirichlet problem (see [9, p. 41])

$$(\Delta_{n-1} + \lambda)\varphi = 0 \quad \text{on} \;\; \Omega,$$
$$\varphi = 0 \quad \text{on} \;\; \partial\Omega.$$

We denote the least positive eigenvalue of this boundary value problem by λ and the normalized positive eigenfunction corresponding to λ by φ,

$$\int_{\Omega} \varphi^2(X) d\Omega = 1,$$

where $X \in \Omega$ and $d\Omega$ is the $(n-1)$-dimensional volume element.

The set

$$\Omega \times \mathbf{R} = \{P = (X, y) \in \mathbf{R}^n; X \in \Omega, y \in \mathbf{R}\}$$

in $\mathbf{R}^n$ is simply denoted by $T_n(\Omega)$. We call it a cylinder (see [3,11,12]). In the following, we denote the sets $\Omega \times I$ and $\partial\Omega \times I$ with an interval I on $\mathbf{R}$ by $T_n(\Omega; I)$ and $S_n(\Omega; I)$ respectively. Hence $S_n(\Omega; \mathbf{R})$ denoted simply by $S_n(\Omega)$ is $\partial T_n(\Omega)$.

In order to make the subsequent consideration simpler, we put a rather strong assumption on Ω throughout this paper: if $n \geq 3$, then Ω is a $C^{2,\alpha}$-domain $(0 < \alpha < 1)$ in $\mathbf{R}^{n-1}$ surrounded by a finite number of mutually disjoint closed hypersurfaces (e.g. see [4, p. 88–89] for the definition of $C^{2,\alpha}$-domain).

Let $\mathcal{G}_\Omega(P, Q)$ be the Green function of $T_n(\Omega)$ $(P, Q \in T_n(\Omega))$. Then the ordinary Poisson kernel in $T_n(\Omega)$ is defined by

$$\mathcal{PI}_\Omega(P, Q) = \frac{1}{c_n} \frac{\partial \mathcal{G}_\Omega(P, Q)}{\partial n_Q},$$

where $\partial/\partial n_Q$ denotes the differentiation at $Q \in S_n(\Omega)$ along the inward normal into $T_n(\Omega)$ for any $P \in T_n(\Omega)$. Here, $c_2 = 2$ and $c_n = (n-2)w_n$ when $n \geq 3$, where w_n is the surface area of the unit sphere in $\mathbf{R}^n$. It follows from our assumption on Ω that $\mathcal{PI}_\Omega(P, Q)$ is continuous on $S_n(\Omega)$ (see [4, Th. 6.15]).

The Poisson integral $\mathcal{PI}_\Omega[g](P)$ of g in $T_n(\Omega)$ is defined as follows

$$\mathcal{PI}_\Omega[g](P) = \int_{S_n(\Omega)} \mathcal{PI}_\Omega(P, Q) g(Q) d\sigma_Q,$$

where $g(Q)$ is a locally integrable function on $S_n(\Omega)$ and $d\sigma_Q$ is the surface area element on $S_n(\Omega)$.

Let $h(P)$ be a function in $T_n(\Omega)$, we use the stand notations $h^+ = \max\{h, 0\}$ and $h^- = -\min\{h, 0\}$. The integral