

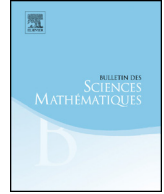


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# Asymptotic behavior of Poisson integrals in a cylinder and its application to the representation of harmonic functions<sup>☆</sup>



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## ABSTRACT

Our first aim in this paper is to deal with asymptotic behavior of Poisson integrals in a cylinder. Next Carleman's formula for harmonic functions in it is also proved. As an application of them, we finally give the integral representation of harmonic functions in a cylinder.

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## 1. Introduction and main results

Let  $\mathbf{R}$  be the set of all real numbers. The boundary and the closure of a set  $E$  in  $n$ -dimensional Euclidean space  $\mathbf{R}^n$  ( $n \geq 2$ ) are denoted by  $\partial E$  and  $\overline{E}$  respectively. For positive functions  $h_1$  and  $h_2$ , we say that  $h_1 \lesssim h_2$  if  $h_1 \leq ch_2$  for some constant  $c > 0$ . If  $h_1 \lesssim h_2$  and  $h_2 \lesssim h_1$ , then we say that  $h_1 \approx h_2$ .

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Let  $\Delta_n$  be the Laplace operator and  $\Omega$  be a bounded domain in  $\mathbf{R}^{n-1}$  with smooth boundary  $\partial\Omega$ . Consider the Dirichlet problem (see [9, p. 41])

$$\begin{aligned} (\Delta_{n-1} + \lambda)\varphi &= 0 \quad \text{on } \Omega, \\ \varphi &= 0 \quad \text{on } \partial\Omega. \end{aligned}$$

We denote the least positive eigenvalue of this boundary value problem by  $\lambda$  and the normalized positive eigenfunction corresponding to  $\lambda$  by  $\varphi$ ,

$$\int_{\Omega} \varphi^2(X) d\Omega = 1,$$

where  $X \in \Omega$  and  $d\Omega$  is the  $(n - 1)$ -dimensional volume element.

The set

$$\Omega \times \mathbf{R} = \{P = (X, y) \in \mathbf{R}^n; X \in \Omega, y \in \mathbf{R}\}$$

in  $\mathbf{R}^n$  is simply denoted by  $T_n(\Omega)$ . We call it a cylinder (see [3,11,12]). In the following, we denote the sets  $\Omega \times I$  and  $\partial\Omega \times I$  with an interval  $I$  on  $\mathbf{R}$  by  $T_n(\Omega; I)$  and  $S_n(\Omega; I)$  respectively. Hence  $S_n(\Omega; \mathbf{R})$  denoted simply by  $S_n(\Omega)$  is  $\partial T_n(\Omega)$ .

In order to make the subsequent consideration simpler, we put a rather strong assumption on  $\Omega$  throughout this paper: if  $n \geq 3$ , then  $\Omega$  is a  $C^{2,\alpha}$ -domain ( $0 < \alpha < 1$ ) in  $\mathbf{R}^{n-1}$  surrounded by a finite number of mutually disjoint closed hypersurfaces (e.g. see [4, p. 88–89] for the definition of  $C^{2,\alpha}$ -domain).

Let  $\mathcal{G}_{\Omega}(P, Q)$  be the Green function of  $T_n(\Omega)$  ( $P, Q \in T_n(\Omega)$ ). Then the ordinary Poisson kernel in  $T_n(\Omega)$  is defined by

$$\mathcal{P}\mathcal{I}_{\Omega}(P, Q) = \frac{1}{c_n} \frac{\partial \mathcal{G}_{\Omega}(P, Q)}{\partial n_Q},$$

where  $\partial/\partial n_Q$  denotes the differentiation at  $Q \in S_n(\Omega)$  along the inward normal into  $T_n(\Omega)$  for any  $P \in T_n(\Omega)$ . Here,  $c_2 = 2$  and  $c_n = (n - 2)w_n$  when  $n \geq 3$ , where  $w_n$  is the surface area of the unit sphere in  $\mathbf{R}^n$ . It follows from our assumption on  $\Omega$  that  $\mathcal{P}\mathcal{I}_{\Omega}(P, Q)$  is continuous on  $S_n(\Omega)$  (see [4, Th. 6.15]).

The Poisson integral  $\mathcal{P}\mathcal{I}_{\Omega}[g](P)$  of  $g$  in  $T_n(\Omega)$  is defined as follows

$$\mathcal{P}\mathcal{I}_{\Omega}[g](P) = \int_{S_n(\Omega)} \mathcal{P}\mathcal{I}_{\Omega}(P, Q)g(Q)d\sigma_Q,$$

where  $g(Q)$  is a locally integrable function on  $S_n(\Omega)$  and  $d\sigma_Q$  is the surface area element on  $S_n(\Omega)$ .

Let  $h(P)$  be a function in  $T_n(\Omega)$ , we use the stand notations  $h^+ = \max\{h, 0\}$  and  $h^- = -\min\{h, 0\}$ . The integral

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