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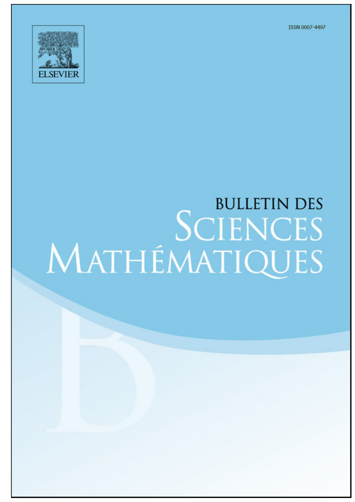

A fundamental differential system of 3-dimensional Riemannian geometry

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Abstract

We briefly recall a fundamental exterior differential system of Riemannian geometry and apply it to the case of three dimensions. Here we find new global tensors and intrinsic invariants of oriented Riemaniann 3-manifolds. In particular, we develop the study of $\nabla\mathrm{Ric}$. The exterior differential system leads to a remarkable Weingarten type equation for immersed surfaces in hyperbolic 3-space. A new independent proof for low dimensions of the structural equations gives new insight on the intrinsic exterior differential system.

Key Words: tangent sphere bundle, Riemaniann metric, structure group, Euler-Lagrange system, 3-manifold.

MSC 2010: Primary: 58A32; Secondary: 58A15, 53C20, 53C28

1 A fundamental differential system

This article presents the fundamental exterior differential system of Riemannian geometry introduced in [7], now developed on the 3-dimensional case.

The intrinsic structure found in [7] consists, in general, on a natural set of differential forms $\alpha_0,\ldots,\alpha_n$ existing on the total space $\mathcal{S}$ of the unit tangent sphere bundle $SM \longrightarrow M$ of any given oriented Riemannian $n+1$-manifold M. It is well-known that $\mathcal{S}$ is a contact Riemannian manifold with the Sasaki metric.

The theory applied to Riemannian surfaces is classical, as we shall recall next, considering the case $n=1$. Indeed, the famous structural equations due to Cartan give a global coframing on $\mathcal{S}$, the total space of the tangent circle bundle over a surface M, with contact 1-form θ and two 1-forms α_0 and α_1. Denoting by c the Gauss curvature of M, we find the following equations e.g. in [22, pp. 168–169]:

$$\begin{gathered} \mathrm{d}\theta = \alpha_1\wedge\alpha_0, \\ \mathrm{d}\alpha_0 = \theta\wedge\alpha_1 \qquad \mathrm{d}\alpha_1 = c\,\alpha_0\wedge\theta. \end{gathered} \tag{1}$$