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Partial differential equations/Numerical analysis

Transmission eigenvalues with artificial background for explicit material index identification

Une identification d'indice explicite au moyen de valeurs propres de transmission pour un milieu de référence artificiel

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ABSTRACT

We are interested in the problem of retrieving information on the refractive index n of a penetrable inclusion embedded in a reference medium from farfield data associated with incident plane waves. Our approach relies on the use of transmission eigenvalues (TEs) that carry information on n and that can be determined from the knowledge of the farfield operator F. In this note, we explain how to modify F into a farfield operator $F^{\text{art}} = F - \tilde{F}$, where $\tilde{F}$ is computed numerically, corresponding to well-chosen artificial background and for which the associated TEs provide more accessible information on n.

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RÉSUMÉ

Nous souhaitons retrouver l'indice n d'une inclusion pénétrable dans un milieu de référence connu à partir de la donnée de champs lointains associés à des ondes planes incidentes. Pour ce faire, nous utilisons les valeurs propres de transmission (VPT) qui dépendent de n et qui peuvent être déterminées à partir de l'opérateur de champ lointain F. Dans cette note, nous expliquons comment modifier F en un opérateur de champ lointain $F^{\text{art}} = F - \tilde{F}$, où $\tilde{F}$ est calculé numériquement, correspondant à un milieu de référence artificiel et pour lequel les VPT associées fournissent une information plus directe sur n.

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1. Introduction

In recent years, sampling methods offered different perspectives in solving time harmonic inverse scattering problems [6]. In addition to allow for a non-iterative scheme to retrieve the support of inhomogeneities from multistatic data, these methods revealed the possibility to construct from the data a spectrum related to the material properties. This spectrum

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corresponds to the set of Transmission Eigenvalues (TEs) of the so-called Interior Transmission Problem (ITP) (see (4)). In the justification of sampling methods, substantial efforts have been made to prove discreteness of the set of TEs [7], because most of these methods fail at frequencies corresponding to these values. However, since the work in [5], exploiting the failure of the reconstruction methods at TEs, it was proved that they can be determined from measured data and therefore can be exploited to infer information on the material properties. The determination of TEs from measured data has been improved using the framework of the Generalized Linear Sampling Method (GLSM) where exact knowledge of the support is no longer needed [2,6]. See also [14] for a different approach.

Under certain assumptions on n, the refractive index of the considered inhomogeneity appearing in Problem (1) below, it has been proved that there exist an infinite number of real positive TEs ($k^2 > 0$) [8]. Note that in practice only real positive TEs are of interest because one can only play with real wavenumbers for measurements. This result of existence of real positive TEs is not obvious because the ITP (see equation after (4) below) is quadratic in k^2 and it does not seem possible to see the spectrum of (4) as the spectrum of a self-adjoint operator. In particular, in 1D situations, it has been established that complex TEs do exist.

Although mathematically interesting, relying on transmission eigenvalues to determine quantitative features on n is difficult. The reason is twofold. First, information is lost in complex eigenvalues which cannot be measured in practice. Second it is difficult to establish sharp estimates for real TEs with respect to n due to the complexity of the problem. In this note, we explain how to work with another farfield operator F^{art} corresponding to an artificial background (reference medium) for which the associated TEs have a more direct connection with n. Put differently, working with F^{art}, our goal is to simplify the solution to the inverse spectral problem consisting in determining n from the knowledge of real positive TEs. Important in the analysis is the fact that F^{art} is given by the formula $F^{\text{art}} = F - \tilde{F}$ where $\tilde{F}$ can be obtained via a rather direct numerical computation. Therefore, in practice F^{art} can also be considered as a data. Interestingly also, our approach does not require a priori knowledge of the exact support of the inhomogeneity. It is sufficient to know that the defect in the reference medium is located in a given bounded region.

Close to our study are the papers [11,9,3]. In the first one, the authors reformulate the ITP as an eigenvalue problem for the material coefficient. In the second and third ones, it is explained how to identify n from the knowledge of $F(k) - \tilde{F}(k,\gamma)$ at a single wavenumber k and for a range of γ. Here $F(k) - \tilde{F}(k,\gamma)$ can be seen as the farfield operator corresponding to a background depending on an artificial parameter γ. In comparison with our approach, this method is interesting because it requires to know F at a single wavenumber ($\tilde{F}(k,\gamma)$ can be computed numerically). However, the relation between associated TEs and n is a bit more complex than in our case.

2. Setting

We assume that the propagation of waves in time harmonic regime in the reference medium $\mathbb{R}^d$, $d = 2, 3$, is governed by the Helmholtz equation $\Delta u + k^2 u = 0$, with $k > 0$ being the wavenumber. The localized perturbation in the reference medium is modeled by some bounded open set $\Omega \subset \mathbb{R}^d$ with Lipschitz boundary $\partial\Omega$ and a refractive index $n \in \mathrm{L}^\infty(\mathbb{R}^d)$. We assume that n is real valued, that $n = 1$ in $\mathbb{R}^d \setminus \Omega$ and that $\operatorname{ess\,inf}_\Omega n$ is positive. The scattering of the incident plane wave $u_i(\cdot, \boldsymbol{\theta}_i) := \mathrm{e}^{\mathrm{i}k\boldsymbol{\theta}_i\cdot\boldsymbol{x}}$ of direction of propagation $\boldsymbol{\theta}_i \in \mathbb{S}^{d-1}$ by Ω is described by the problem

$$\left|\begin{array}{l}\text{find } u = u_i + u_s \text{ such that}\\ \Delta u + k^2 n\, u \;\;=\;\; 0 \quad \text{in } \mathbb{R}^d,\\ \displaystyle\lim_{r\to+\infty} r^{\frac{d-1}{2}}\left(\frac{\partial u_s}{\partial r} - \mathrm{i}ku_s\right) = 0,\end{array}\right. \tag{1}$$

with $u_i = u_i(\cdot, \boldsymbol{\theta}_i)$. The last line of (1), where $r = |\boldsymbol{x}|$, is the Sommerfeld radiation condition and is assumed to hold uniformly with respect to $\boldsymbol{\theta}_s = \boldsymbol{x}/r$. For all $k > 0$, Problem (1) has a unique solution $u \in \mathrm{H}^2_{\text{loc}}(\mathbb{R}^d)$. The scattered field $u_s(\cdot, \boldsymbol{\theta}_i)$ has the expansion

$$u_s(\boldsymbol{x}, \boldsymbol{\theta}_i) = \mathrm{e}^{\mathrm{i}kr} r^{-\frac{d-1}{2}}\left(u_s^\infty(\boldsymbol{\theta}_s, \boldsymbol{\theta}_i) + O(1/r)\right), \tag{2}$$

as $r \to +\infty$, uniformly in $\boldsymbol{\theta}_s \in \mathbb{S}^{d-1}$. The function $u_s^\infty(\cdot, \boldsymbol{\theta}_i) : \mathbb{S}^{d-1} \to \mathbb{C}$ is called the farfield pattern associated with $u_i(\cdot, \boldsymbol{\theta}_i)$. From the farfield pattern, we can define the farfield operator $F : \mathrm{L}^2(\mathbb{S}^{d-1}) \to \mathrm{L}^2(\mathbb{S}^{d-1})$ such that

$$(Fg)(\boldsymbol{\theta}_s) = \int_{\mathbb{S}^{d-1}} g(\boldsymbol{\theta}_i)\, u_s^\infty(\boldsymbol{\theta}_s, \boldsymbol{\theta}_i)\, \mathrm{d}s(\boldsymbol{\theta}_i). \tag{3}$$

The function Fg corresponds to the farfield pattern for the scattered field in (1) with $u_i = u_i(g) := \int_{\mathbb{S}^{d-1}} g(\boldsymbol{\theta}_i)\mathrm{e}^{\mathrm{i}k\boldsymbol{\theta}_i\cdot\boldsymbol{x}}\, \mathrm{d}s(\boldsymbol{\theta}_i)$ (Herglotz wave function). Define the operator $\mathcal{H} : \mathrm{L}^2(\mathbb{S}^{d-1}) \to \mathrm{L}^2(\Omega)$ such that $\mathcal{H}g = u_i(g)|_\Omega$ and the space $\mathrm{H}_{\text{inc}}(\Omega) := \{v \in \mathrm{L}^2(\Omega);\; \Delta v + k^2 v = 0 \text{ in } \Omega\}$. It is known that $\mathrm{H}_{\text{inc}}(\Omega)$ is nothing but the closure of the range of the operator $\mathcal{H}$ in $\mathrm{L}^2(\Omega)$. Observing that $\Delta u_s + k^2 n u_s = k^2(1-n)u_i(g)$ (in particular u_s depends only on the values of $u_i(g)|_\Omega$), we can factorize F as $F = G\mathcal{H}$ where the operator $G : \mathrm{H}_{\text{inc}}(\Omega) \to \mathrm{L}^2(\mathbb{S}^{d-1})$ is the extension by continuity of the mapping $u_i(g)|_\Omega \mapsto u_s^\infty$. The

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