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Ordinary differential equations

Existence and concentration result for a class of fractional Kirchhoff equations with Hartree-type nonlinearities and steep potential well ☆

Résultats d'existence et de concentration pour une classe d'équations de Kirchhoff fractionnaires avec non-linéarité de type Hartree and puits de potentiel abrupt

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ABSTRACT

In this paper, we study the following fractional Kirchhoff equations

$$\begin{cases}(a+b\int_{\mathbb{R}^N}|(-\triangle)^{\frac{\alpha}{2}}u|^2\mathrm{d}x)(-\triangle)^{\alpha}u+\lambda V(x)u=(|x|^{-\mu}*G(u))g(u),\\ u\in H^{\alpha}(\mathbb{R}^N),\ N\geq 3,\end{cases}$$

where $a, b > 0$ are constants, and $(-\triangle)^{\alpha}$ is the fractional Laplacian operator with $\alpha \in (0,1)$, $2 < 2^*_{\alpha,\mu} = \frac{2N-\mu}{N-2\alpha} \leq 2^*_{\alpha} = \frac{2N}{N-2\alpha}$, $0 < \mu < 2\alpha$, $\lambda > 0$, is real parameter. 2^*_{α} is the critical Sobolev exponent. g satisfies the Berestycki–Lions-type condition (see [2]). By using Pohožaev identity and concentration-compact theory, we show that the above problem has at least one nontrivial solution. Furthermore, the phenomenon of concentration of solutions is also explored. Our result supplements the results of Lü (see [8]) concerning the Hartree-type nonlinearity $g(u) = |u|^{p-1}u$ with $p \in (2, 6-\alpha)$.

RÉSUMÉ

Dans ce texte, nous étudions les équations de Kirchhoff fractionnaires suivantes :

$$\begin{cases}(a+b\int_{\mathbb{R}^N}|(-\triangle)^{\frac{\alpha}{2}}u|^2\mathrm{d}x)(-\triangle)^{\alpha}u+\lambda V(x)u=(|x|^{-\mu}*G(u))g(u),\\ u\in H^{\alpha}(\mathbb{R}^N),\ N\geq 3,\end{cases}$$

où $a, b > 0$ sont des constantes et $(-\triangle)^{\alpha}$ est l'opérateur laplacien fractionnaire avec $\alpha \in (0,1)$, $2 < 2^*_{\alpha,\mu} = \frac{2N-\mu}{N-2\alpha} \leq 2^*_{\alpha} = \frac{2N}{N-2\alpha}$, $0 < \mu < 2\alpha$ et $\lambda > 0$ des paramètres réels. Ici, 2^*_{α} désigne l'exposant de Sobolev critique et g satisfait une condition de type Berestycki–Lions (voir [2]). En utilisant l'identité de Pohozaev et la théorie de concentration–compacité, nous

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montrons que le problème ci-dessus a au moins une solution non triviale. De plus, nous explorons le phénomène de concentration des solutions. Nos résultats complètent ceux de Lü (voir [8]) sur la non-linéarité de type Hartree $g(u) = |u|^{p-1}$, avec $p \in (2, 6-\alpha)$.

1. Introduction and main results

In this paper, we consider the following fractional Kirchhoff equations

$$\begin{cases} (a+b\int_{\mathbb{R}^N}|(-\triangle)^{\frac{\alpha}{2}}u|^2\mathrm{d}x)(-\triangle)^\alpha u+\lambda V(x)u=(|x|^{-\mu}*G(u))g(u), \\ u\in H^\alpha(\mathbb{R}^N),\ N\ge 3, \end{cases} \tag{1.1}$$

where $a, b > 0$ are constants, and $\alpha \in (0, 1)$, $(-\triangle)^\alpha$ stands for the fractional Laplacian operator, which is defined by $(-\triangle)^\alpha u = C(N,\alpha)P.V.\int_{\mathbb{R}^N\setminus B_\varepsilon(x)}\frac{u(x)-u(y)}{|x-y|^{N+2\alpha}}\mathrm{d}y,\ \ x\in\mathbb{R}^N$, where $P.V.$ is used as abbreviation for 'in the sense of principal value' and $C(N,\alpha)$ is a suitable positive normalization constant. In fact, problem (1.1) is a fractional version of a model, the so-called Kirchhoff equation, introduced by Kirchhoff [1]. More precisely, Kirchhoff established a model given by the equation

$$\rho\frac{\partial^2 u}{\partial t^2}-(\frac{\rho_0}{h}+\frac{E}{2L}\int_0^L|\frac{\partial u}{\partial x}|^2\mathrm{d}x)\frac{\partial^2 u}{\partial x^2}=0, \tag{1.2}$$

where ρ, ρ_0, h, E and L are constants, which extends the classical D'Alembert wave equation by considering the effects of the changes in the length of the string the vibrations. In particular, the Kirchhoff equation (1.2) contains a nonlocal coefficient $\rho_0/h+(E/2L)\int_0^L|\frac{\partial u}{\partial x}|^2\mathrm{d}x$, which depends on the average $(1/L)\int_0^L|\frac{\partial u}{\partial x}|^2\mathrm{d}x$ of the kinetic energy $|\frac{\partial u}{\partial x}|^2$ on $[0, L]$, and hence the equation is no longer a pointwise identity. Moreover, nonlocal boundary problems like equation (1.2) can be used to model several physical and biological systems where u describes a process that depends on the average of itself, such as the population density, see [10]. We point out that (1.2) received much attention only after Lions [3] introduced an abstract framework to this problem. For more mathematical and physical background of the fractional Kirchhoff problem (1.1), we refer the reader to the papers [1,3] and to the references.

Recently, there are many papers (see [8,11,12]) studying the Kirchoff-type problem with Hartree-type nonlinearity. To our best knowledge, for a Hartree-type nonlinearity term $g(u) = |u|^p$ with $p \in (1, 2]$, there is still no result. Inspired by the above facts, in this paper, our result supplements the results of Lü (see [8]) concerning the Hartree-type nonlinearity term $g(u) = |u|^{p-1}u$ with $p \in (2, 6-\alpha)$.

Before stating our main results, we give the following assumption on $V(x)$ and $g(t)$.

(V_1) $V \in C(\mathbb{R}^N, \mathbb{R})$ and $V(x) \ge 0$ on $\mathbb{R}^N$, and satisfies $V(x) - (\nabla V(x), x) > 0$.

(V_2) There is $M > 0$ such that $V := \{x \in \mathbb{R}^N | V(x) < M\}$ has finite measure.

(V_3) $\Omega = \mathrm{int}\{V^{-1}(0)\}$ is nonempty and has smooth boundary $\partial\Omega$.

(g_1) There exists $C > 0$ such that for every $t \in \mathbb{R}$, $|\mathrm{tg}(t)| \le C(|t|^2 + |t|^{\frac{2N-\mu}{N-2\alpha}})$.

(g_2) Let $G : t \in \mathbb{R} \mapsto \int_0^t g(t)\,\mathrm{d}\tau$ and assume that $\lim\limits_{t\to 0}\frac{G(t)}{|t|^2} = 0$ and $\lim\limits_{t\to\infty}\frac{G(t)}{|t|^{\frac{2N-\mu}{N-2\alpha}}} = 0$.

(g_3) There exists $t_0 \in \mathbb{R}$ such that $G(t_0) \ne 0$.

This kind of hypotheses was first introduced by Bartsch and Wang [9] in the study of a nonlinear Schrödinger equation and the potential $\lambda V(x)$ with V satisfying (V_1)–(V_3) is referred to as the steep well potential whose depth is controlled by the parameter λ.

Now we state our main results.

Theorem 1.1. *Under assumptions $0 < \mu < 2\alpha$, (V_1)–(V_3) and (g_1)–(g_3), the system (1.1) has for any $\lambda > 0$ at least one nontrivial solution u_λ. Moreover $u_\lambda \to u_0$ as $\lambda \to \infty$, and u_0 is the ground state solution to*

$$(a+b\int_\Omega|(-\triangle)^{\frac{\alpha}{2}}u|^2\mathrm{d}x)(-\triangle)^\alpha u=(|x|^{-\mu}*G(u))g(u). \tag{1.3}$$

Remark 1.2. Note that in the case where $g(u) = |u|^{p-1}u$, our conditions (g_1)–(g_3) cover the full subcritical range of $p \in (1, 2]$.

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