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Combinatorics/Ordinary differential equations

Correlation between Adomian and partial exponential Bell polynomials

Corrélation des polynômes d'Adomian et des polynômes de Bell exponentiels partiels

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A R T I C L E I N F O

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A B S T R A C T

We obtain some recurrence relationships among the partition vectors of the partial exponential Bell polynomials. On using such results, the n-th Adomian polynomial for any nonlinear operator can be expressed explicitly in terms of the partial exponential Bell polynomials. Some new identities for the partial exponential Bell polynomials are obtained by solving certain ordinary differential equations using the Adomian decomposition method.

R É S U M É

Nous montrons des relations de récurrence entre les vecteurs partition des polynômes de Bell exponentiels partiels. Utilisant ces relations, le n-ième polynôme d'Adomian, pour n'importe quel opérateur non linéaire, s'exprime explicitement en termes des polynômes de Bell exponentiels partiels. On en déduit des identités nouvelles pour ces derniers, via la solution de certaines équations différentielles ordinaires, en utilisant la méthode de décomposition d'Adomian.

1. Introduction

The Bell polynomials studied by Bell [4,5] are special polynomials in combinatorial analysis, with numerous applications in different areas of mathematics. The incomplete or partial exponential Bell polynomials $B_{n,k}$ (see [7], [10, p. 96]) in $n-k+1$ variables are triangular arrays of polynomials defined by

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$$B_{n,k}(u_1, u_2, \ldots, u_{n-k+1}) = n! \sum_{\Lambda_n^k} \prod_{j=1}^{n-k+1} \frac{1}{k_j!} \left(\frac{u_j}{j!} \right)^{k_j}, \tag{1.1}$$

where the partition set is given by $\Lambda_n^k = \left\{ (k_1, k_2, \ldots, k_{n-k+1}) : \sum_{j=1}^{n-k+1} k_j = k, \sum_{j=1}^{n-k+1} jk_j = n, k_j \in \mathbb{N}_0 \right\}$. Here $\mathbb{N}_m = \{x : x \geq m, x \in \mathbb{N} \cup \{0\}\}$ and $\mathbb{N}$ denotes the set of positive integers. Also, the sum

$$B_n(u_1, u_2, \ldots, u_n) = \sum_{k=1}^{n} B_{n,k}(u_1, u_2, \ldots, u_{n-k+1}), \tag{1.2}$$

is called the n-th complete exponential Bell polynomial. For more details, results and some known identities on Bell polynomials, we refer the reader to [6, pp. 133–137], [7] and [10, pp. 95–98].

Next we briefly explain the Adomian decomposition method (ADM) [2,3], which will be used later to obtain some new identities for Bell polynomials. In ADM, the solution to the functional equation

$$u = f + L(u) + N(u), \tag{1.3}$$

where L and N are linear and nonlinear operators respectively and f is a known function, is expressed in the form of an infinite series

$$u = \sum_{n=0}^{\infty} u_n. \tag{1.4}$$

The nonlinear term $N(u)$ decomposes as

$$N(u) = \sum_{n=0}^{\infty} A_n(u_0, u_1, \ldots, u_n), \tag{1.5}$$

where A_n denotes the n-th Adomian polynomial in $u_0, u_1, \ldots, u_n$. Also, the series (1.4) and (1.5) are assumed to be absolutely convergent. So, (1.3) can be rewritten as

$$\sum_{n=0}^{\infty} u_n = f + \sum_{n=0}^{\infty} L(u_n) + \sum_{n=0}^{\infty} A_n.$$

Thus the u_ns are obtained by the following recursive relation

$$u_0 = f \quad \text{and} \quad u_n = L(u_{n-1}) + A_{n-1}.$$

The crucial step involved in ADM is the calculation of Adomian polynomials. Adomian [2, pp. 19–21] gave a method for determining these polynomials, by parameterizing u as $u_\lambda = \sum_{n=0}^{\infty} u_n \lambda^n$ and assuming $N(u_\lambda)$ to be analytic in λ, which decomposes as $N(u_\lambda) = \sum_{n=0}^{\infty} A_n(u_0, u_1, \ldots, u_n)\lambda^n$. Hence, Adomian polynomials are given by

$$A_n(u_0, u_1, \ldots, u_n) = \frac{1}{n!} \frac{\partial^n N(u_\lambda)}{\partial \lambda^n} \bigg|_{\lambda=0}, \ \forall \, n \in \mathbb{N}_0. \tag{1.6}$$

An improved version of the above result (see Zhu et al. [13]) is given by

$$A_n(u_0, u_1, \ldots, u_n) = \frac{1}{n!} \frac{\partial^n N(\sum_{k=0}^{n} u_k \lambda^k)}{\partial \lambda^n} \bigg|_{\lambda=0}, \ \forall \, n \in \mathbb{N}_0. \tag{1.7}$$

Rach [12] suggested the following formula for these polynomials: $A_0(u_0) = N(u_0)$ and

$$A_n(u_0, u_1, \ldots, u_n) = \sum_{k=1}^{n} C(k,n) N^{(k)}(u_0), \ \forall \, n \in \mathbb{N}, \tag{1.8}$$

where

$$C(k,n) = \sum_{\Theta_n^k} \prod_{j=1}^{n} \frac{u_j^{k_j}}{k_j!}, \tag{1.9}$$

and the summation is taken over the partition set $\Theta_n^k = \left\{ (k_1, k_2, \ldots, k_n) : \sum_{j=1}^{n} k_j = k, \sum_{j=1}^{n} jk_j = n, k_j \in \mathbb{N}_0 \right\}$. Also, $N^{(k)}(.)$ denotes the k-th derivative of the nonlinear term. One can easily show the equivalence of (1.6) and (1.8) using the Faà