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Mathematical analysis

Circumventing the lack of dissipation in certain Oldroyd models

Comment contourner le manque de dissipation de certains modèles de Oldroyd

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A R T I C L E I N F O

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A B S T R A C T

We modify an argument of Renardy proving existence and regularity for a subset of a class of models of non-Newtonian fluids suggested by Oldroyd, including the upper-convected and lower-convected Maxwellian models. We suggest an effective method for solving these models, which can provide a variational formulation suitable for finite element computation.

R É S U M É

Nous modifions le raisonnement utilisé par Renardy pour prouver l'existence et la régularité de solutions d'une sous-classe de modèles de fluides non newtoniens introduits par Oldroyd, comme les modèles maxwelliens de sur-convection et sous-convection. Nous proposons une méthode itérative variationnelle de calcul de solutions qui s'adapte aux éléments finis.

1. Introduction

We summarize here results obtained in [7] regarding models for non-Newtonian fluids that are a subset of the Oldroyd models [9], including the upper-convected and lower-convected Maxwellian models. The subset we study involves three parameters, the fluid kinematic viscosity η and two rheological parameters λ_1 and μ_1. We refer to this subset as the "three-parameter" subset. We modify the existence proof of Renardy [10] and show that it can be the basis for an effective solution algorithm.

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Well-posedness has also been established [4] for a "five-parameter" subset of the Oldroyd models [9] involving two additional rheological parameters λ_2 and μ_2. The techniques used for these models are quite different from the ones used by Renardy [10] and revisited here. For some reasons explained in [7], we are forced to limit our approach to the three-parameter case. The approaches are complementary, and this potentially reflects significant differences in these models. In [4], $\lambda_2 \neq 0$ is explicitly required, and (as far as we are aware) the bounds obtained would degenerate as $\lambda_2 \to 0$. The condition $\lambda_2 > 0$ leads to an explicit dissipation term that is used in obtaining bounds. When $\lambda_2 = 0$, such explicit dissipation is missing. Thus there is an open question regarding bounds, when $\lambda_2 > 0$, that hold uniformly for λ_2 small.

1.1. Notation

We assume that the fluid domain $\mathcal{D} \subset \mathbb{R}^d$ is connected and has a Lipschitz boundary $\partial\mathcal{D}$. For simplicity, we assume that the boundary conditions on the fluid velocity are Dirichlet: $\mathbf{u} = \mathbf{0}$ on $\partial\mathcal{D}$. We utilize standard Sobolev spaces $W_q^s(\mathcal{D})$ for nonnegative integers s and $1 \le q \le \infty$, consisting of functions whose derivatives of order s or less are in the Lebesgue space $L_q(\mathcal{D})$ [5,1,3]. For vector-valued functions $\mathbf{v}$ and matrix-valued functions $\mathbf{T}$, we will write $\mathbf{v} \in W_q^s(\mathcal{D})^d$ or $\mathbf{T} \in W_q^s(\mathcal{D})^{d^2}$ to indicate that each component of $\mathbf{v}$ or $\mathbf{T}$ is $W_q^s(\mathcal{D})$. We will also write the corresponding norms for vector-valued and tensor-valued functions via

$$\| \mathbf{T} \|_{W_q^s(\mathcal{D})} = \sum_{m=0}^{s} \| |\nabla^m \mathbf{T}| \|_{L_q(\mathcal{D})},$$

where for tensor quantities $\mathbf{T}$ of any order $r \ge 1$, we denote by $|\mathbf{T}|$ the Euclidean norm of $\mathbf{T}$ when viewed as a vector of dimension d^r.

Regarding the regularity of the domain boundary, we make the following assumptions. Consider the elliptic equations

$$v - \Delta v = f \ \text{ in } \mathcal{D}, \qquad \nabla v \cdot \mathbf{n} = 0 \ \text{ on } \partial\mathcal{D}, \tag{1.1}$$

where $\mathbf{n}$ is the unit outer normal to $\partial\mathcal{D}$, and

$$-\Delta v = f \ \text{ in } \mathcal{D}, \qquad v = 0 \ \text{ on } \partial\mathcal{D}. \tag{1.2}$$

We introduce the following condition: suppose that the domain $\mathcal{D}$ has the property that there is a constant C such that each problem (1.1) and (1.2) has a unique solution $v \in H^2(\mathcal{D})$ for all $f \in L_2(\mathcal{D})$ satisfying

$$\| v \|_{H^2(\mathcal{D})} \le C \| f \|_{L_2(\mathcal{D})}. \tag{1.3}$$

Similarly, we consider a Stokes system,

$$-\Delta \mathbf{v} + \nabla p = \mathbf{f} \text{ in } \mathcal{D}, \quad \nabla \cdot \mathbf{v} = 0 \text{ in } \mathcal{D}, \quad \mathbf{v} = \mathbf{0} \text{ on } \partial\mathcal{D}. \tag{1.4}$$

We introduce the following condition: suppose that, for some $q > 1$, the domain $\mathcal{D}$ has the property that there is a constant $C_{q,\mathcal{D}}$ such that, for all $\mathbf{f} \in L_q(\mathcal{D})^d$, there is a unique pair $\mathbf{v} \in W_q^2(\mathcal{D})^d$ and $p \in W_q^1(\mathcal{D})/\mathbb{R}$ solving (1.4) such that

$$\| \mathbf{v} \|_{W_q^2(\mathcal{D})} + \| p \|_{W_q^1(\mathcal{D})/\mathbb{R}} \le C_{q,\mathcal{D}} \| \mathbf{f} \|_{L_q(\mathcal{D})} \ \text{ for all } \mathbf{f} \in L_q(\mathcal{D})^d. \tag{1.5}$$

We assume this holds for all $q \le q_0$ where $q_0 > 1$. Ultimately, many of the results will be restricted to the case $q_0 > d$, where d is the dimension of $\mathcal{D}$.

2. Rheology models

In all (time-independent) models of fluids, the basic equation can be written as

$$\mathbf{u} \cdot \nabla \mathbf{u} + \nabla p = \nabla \cdot \mathbf{T} + \mathbf{f}, \tag{2.6}$$

where $\mathbf{T}$ is called the extra (or deviatoric) stress and $\mathbf{f}$ represents externally given data. The models differ only according to the dependence of the stress on the velocity $\mathbf{u}$.

A three parameter subset of the eight-parameter model of Oldroyd [9] for the extra stress takes the form

$$\mathbf{T} + \lambda_1 (\mathbf{u} \cdot \nabla \mathbf{T} + \mathbf{RT} + \mathbf{TR}^t) - \mu_1 (\mathbf{ET} + \mathbf{TE}) = 2\eta \mathbf{E},$$

where the five parameters λ_2, μ_2, μ_0, ν_0, and ν_1 in [9] are set to zero, and $\mathbf{R} = \frac{1}{2}(\nabla \mathbf{u}^t - \nabla \mathbf{u})$ and $\mathbf{E} = \frac{1}{2}(\nabla \mathbf{u} + \nabla \mathbf{u}^t)$. This can be written equivalently as

$$\mathbf{T} + \lambda_1 (\mathbf{u} \cdot \nabla \mathbf{T} - (\nabla \mathbf{u})\mathbf{T} - \mathbf{T}(\nabla \mathbf{u}^t)) + (\lambda_1 - \mu_1)(\mathbf{ET} + \mathbf{TE}) = 2\eta \mathbf{E}.$$

We can write the full model in the steady case as

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