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Ordinary differential equations

Topological properties of solution sets for partial functional evolution inclusions

Propriétés topologiques des ensembles de solutions d'inclusions fonctionnelles partielles d'évolution

Yong Zhou [a,b], Li Peng [a]

[a] *Faculty of Mathematics and Computational Science, Xiangtan University, Hunan 411105, PR China*
[b] *Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia*

A R T I C L E I N F O

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A B S T R A C T

This paper deals with functional evolution inclusions of neutral type in Banach space when the semigroup is compact as well as noncompact. The topological properties of the solution set is investigated. It is shown that the solution set is nonempty, compact and an R_δ-set which means that the solution set may not be a singleton but, from the point of view of algebraic topology, it is equivalent to a point, in the sense that it has the same homology group as one-point space. As a sample of application, we consider a partial differential inclusion at end of the paper.

R É S U M É

Cette Note traite des inclusions fonctionnelles d'évolution de type neutre dans les espaces de Banach, aussi bien lorsque le semi-groupe est compact que lorsqu'il est non compact. Nous étudions les propriétés topologiques de l'ensemble des solutions. Nous montrons que cet ensemble est non vide, compact, et qu'il est un R_δ-ensemble. Ceci signifie qu'il peut ne pas être réduit à un point, mais qu'il est équivalent, pour la topologie algébrique, à un espace réduit à un point. Plus précisément, l'ensemble des solutions a les mêmes groupes d'homologie qu'un ensemble réduit à un point. Comme exemple d'application, nous considérons enfin une inclusion différentielle partielle.

E-mail address: yzhou@xtu.edu.cn (Y. Zhou).

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1. Introduction

In this paper, we study the following functional evolution inclusion of neutral type

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}[x(t)-h(t,x_t)] \in Ax(t)+F(t,x_t), & t\in[0,b], \\ x(t)=\phi(t), & t\in[-\tau,0], \end{cases} \tag{1.1}$$

where the state $x(\cdot)$ takes value in Banach space X with norm $|\cdot|$, F is a multimap defined on a subset of $[0,b]\times X$, A is the infinitesimal generator of an analytic semigroup $\{T(t)\}_{t\geq 0}$. For any continuous function x defined on $[-\tau,b]$ and any $t\in[0,b]$, we denote by x_t the element of $C([-\tau,0],X)$ defined by $x_t(\theta)=x(t+\theta), \theta\in[-\tau,0]$. Here, $x_t(\cdot)$ represents the history of the state from time $t-\tau$, up to the present time t. For any $c\in C([-\tau,0],X)$ the norm of c is defined by $\|c\|_*=\sup_{\theta\in[-\tau,0]}|c(\theta)|$.

The study of (1.1) is justified and motivated by a neutral partial differential inclusion of parabolic type

$$\begin{cases} \dfrac{\partial}{\partial t}\left(x(t,\xi)-\displaystyle\int_0^{\pi} U(\xi,\zeta)x_t(\theta,\zeta)\,\mathrm{d}\zeta\right) \in \dfrac{\partial^2}{\partial\xi^2}x(t,\xi)+F(t,\xi,x_t(\theta,\xi)), & t\in[0,1],\ \xi\in[0,\pi], \\ x(t,0)=x(t,\pi)=0, & t\in[0,1], \\ x(\theta,\xi)=\phi(\theta)(\xi), & \theta\in[-\tau,0],\ \xi\in[0,\pi], \end{cases}$$

where the functions U and ϕ satisfy appropriate conditions, $F:[0,1]\times[0,\pi]\to 2^{\mathbb{R}}$ is weakly upper semicontinuous with closed convex values.

Particularly, if $h=0$, inclusion (1.1) degenerates to

$$x'(t)\in Ax(t)+F(t,x_t).$$

A strong motivation for investigating this class of inclusions is that a lot of phenomena investigated in hybrid systems with dry friction, processes of controlled heat transfer, obstacle problems and others can be described with the help of various differential inclusions, both linear and nonlinear (cf. [9,16,21]). The theory of differential inclusions is highly developed and constitutes an important branch of nonlinear analysis (see, e.g., Bressan and Wang [7], Donchev et al. [11], Gabor and Quincampoix [14], and the references therein).

Since a differential inclusion usually has many solutions starting at a given point, new issues appear, such as investigation of topological and geometric properties of solution sets, selection of solutions with given properties, evaluation of the reachability sets, etc. An important aspect of topological structure is the R_δ-property, which means that an R_δ-set is acyclic (in particular, nonempty, compact and connected) and may not be a singleton but, from the point of view of algebraic topology, it is equivalent to a point, in the sense that it has the same homology groups as one point space. The topological structure of solution sets of differential inclusions on compact intervals has been investigated intensively by many authors—please see Aronszajn [3], Bothe [6], Deimling [9], Hu and Papageorgiou [15], Staicu [19], and references therein. Moreover, one can find results on the topological structure of solution sets for differential inclusions defined on non-compact intervals (including infinite intervals) from Andres and Pavlačková [2], Andres et al. [1], Bakowska and Gabor [4], Bressan and Wang [7], Chen et al. [8], Gabor and Grudzka [12,13], Gabor and Quincampoix [14], Staicu [20], Wang et al. [22], and references therein.

The researches on the theory for nonlinear evolution inclusion of neutral type are only on their initial stage of development, see [5,17,23]. However, to the best of our knowledge, nothing has been done with the topological properties of solution sets for nonlinear evolution inclusion of neutral type. Our purpose in this paper is to study the topological structure of solution sets for inclusion (1.1).

The paper is organized as follows. In Section 2, we recall some notations, definitions, and preliminary facts from multivalued analysis. Subsection 3.1 is devoted to proving that the solution set for inclusion (1.1) is nonempty compact in the case that the semigroup is compact, then proceed to study the R_δ-set. Subsection 3.2 provides that the solution set for inclusion (1.1) is nonempty compact in the case that the semigroup is noncompact, then proceed to study the R_δ-structure of the solution set of (1.1). Finally, an example is given to illustrate the obtained theory.

2. Preliminaries

In this section, we introduce notations, definitions, and preliminary facts from multivalued analysis which are used throughout this paper.

Let $(X,|\cdot|)$ be a Banach space. $\mathcal{L}(X)$ stands for the space of all linear bounded operators on X with norm $\|\cdot\|$, and $L^1([0,b],X)$ stands for the Banach space consisting of integrable functions from $[0,b]$ to X equipped with the norm

$$\|f\|_1=\int_0^b |f(t)|\,\mathrm{d}t.$$